From Communication Complexity to an Entanglement Spread Area Law

Mehdi Soleimanifar (MIT)

(arxiv: 2004.15009)

Joint work with

Anurag Anshu (UC Berkeley)

Aram Harrow (MIT)

Communication Complexity

Ground State Entanglement

Communication Complexity

Ground State Entanglement

Jointly evaluate $f(x, y)$

Alice x

Bob y

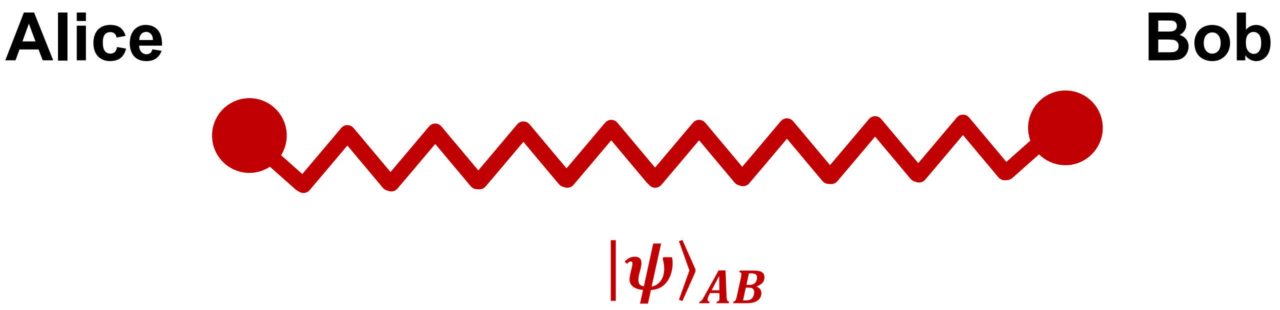
Jointly evaluate $f(x, y)$

Communication Complexity =
Minimum number of exchanged bits to evaluate $f(x, y)$

Jointly perform U_{AB}

Jointly prepare $|\Omega\rangle_{AB}$

Testing Bipartite States



Test whether the shared state is $|\Omega\rangle_{AB}$

Testing Bipartite States

Alice **Bob**

$|\psi\rangle_{AB}$

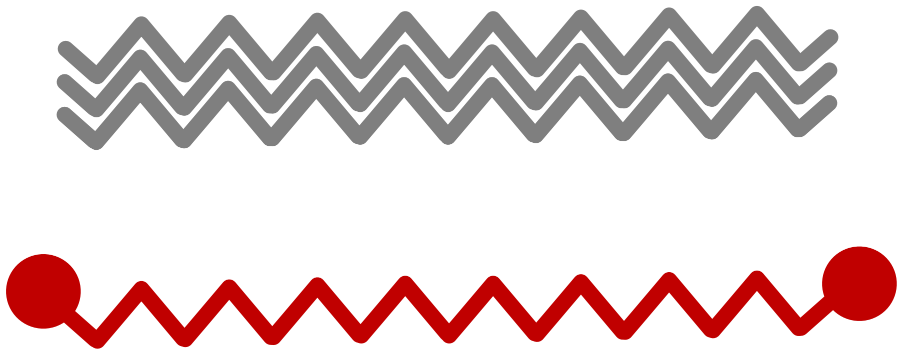
YES NO

$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

Testing Bipartite States

EPR Pairs $|\Phi\rangle_{AB}$



Alice

Bob

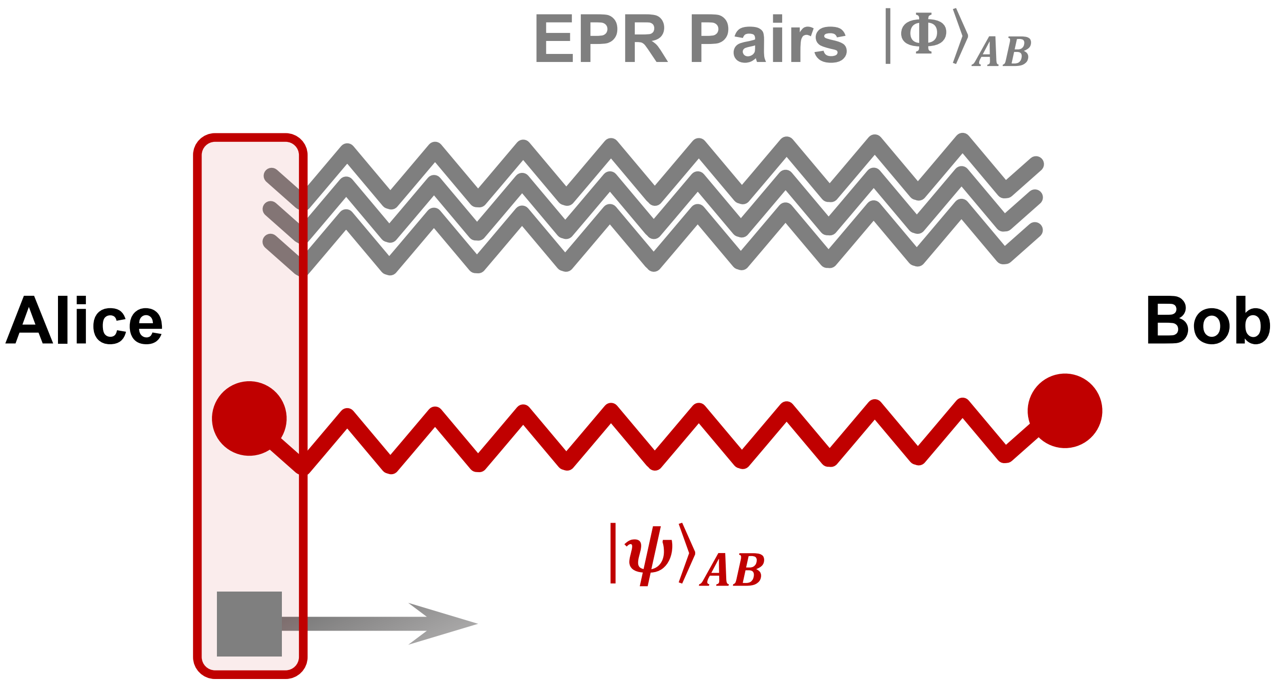
$|\psi\rangle_{AB}$

YES NO

$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

Testing Bipartite States

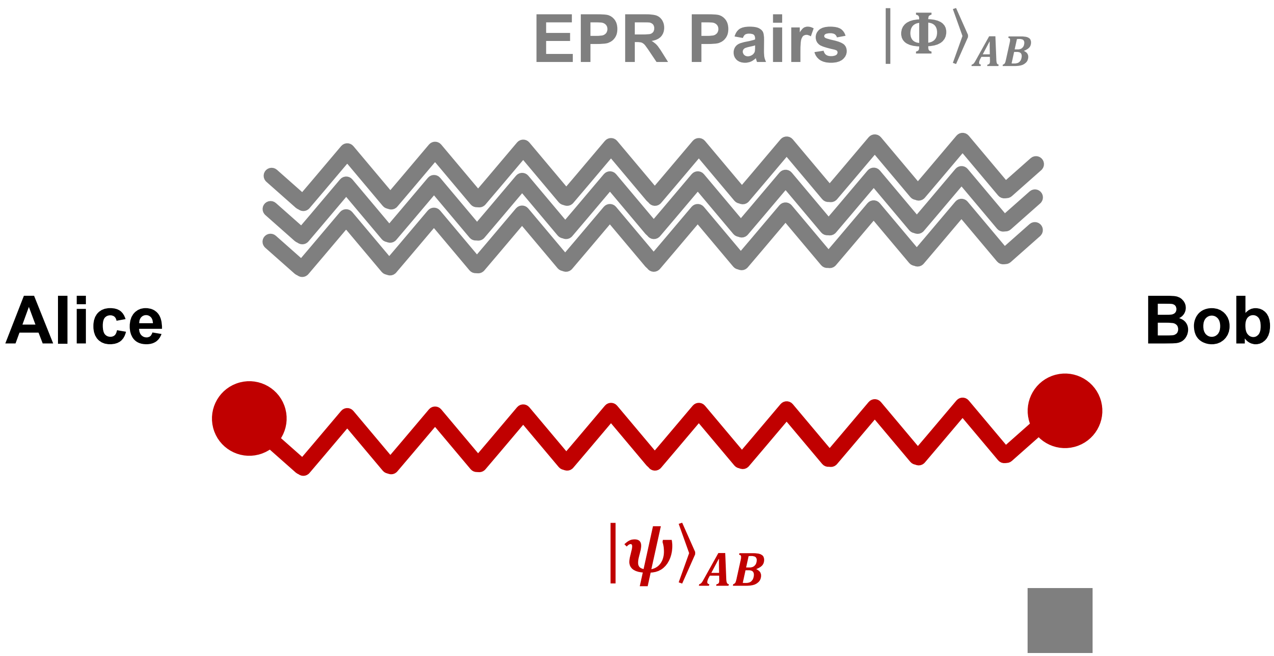


YES NO

$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

Testing Bipartite States

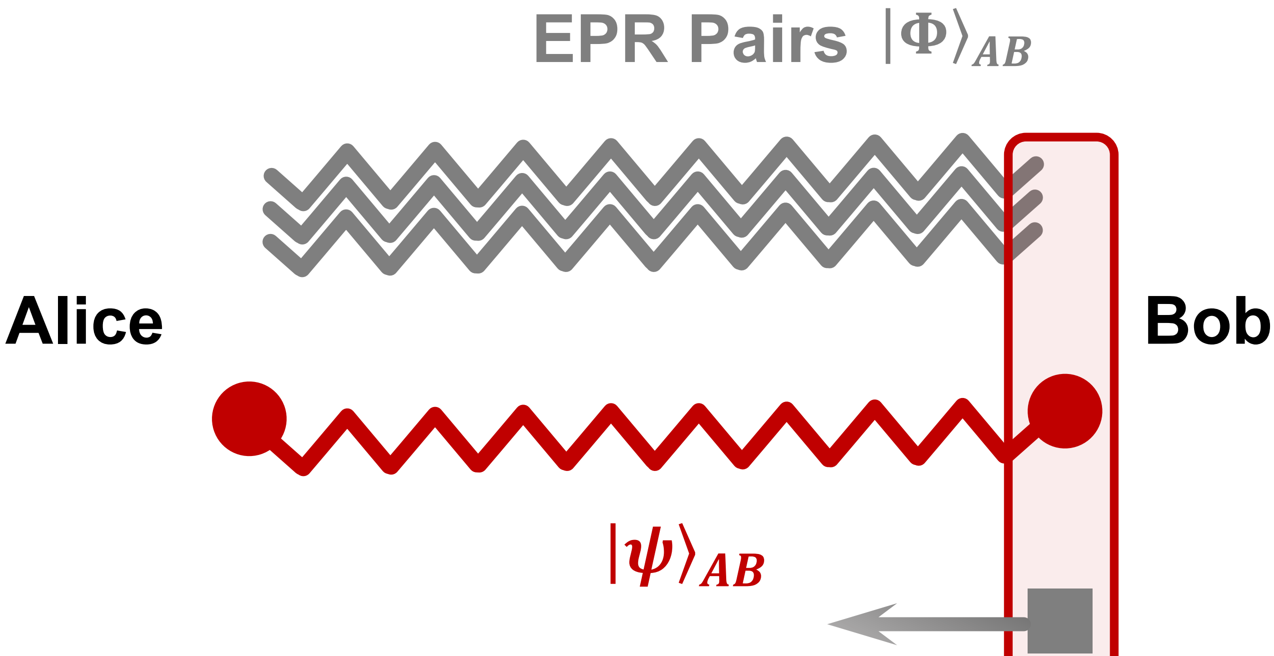


YES NO

$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

Testing Bipartite States

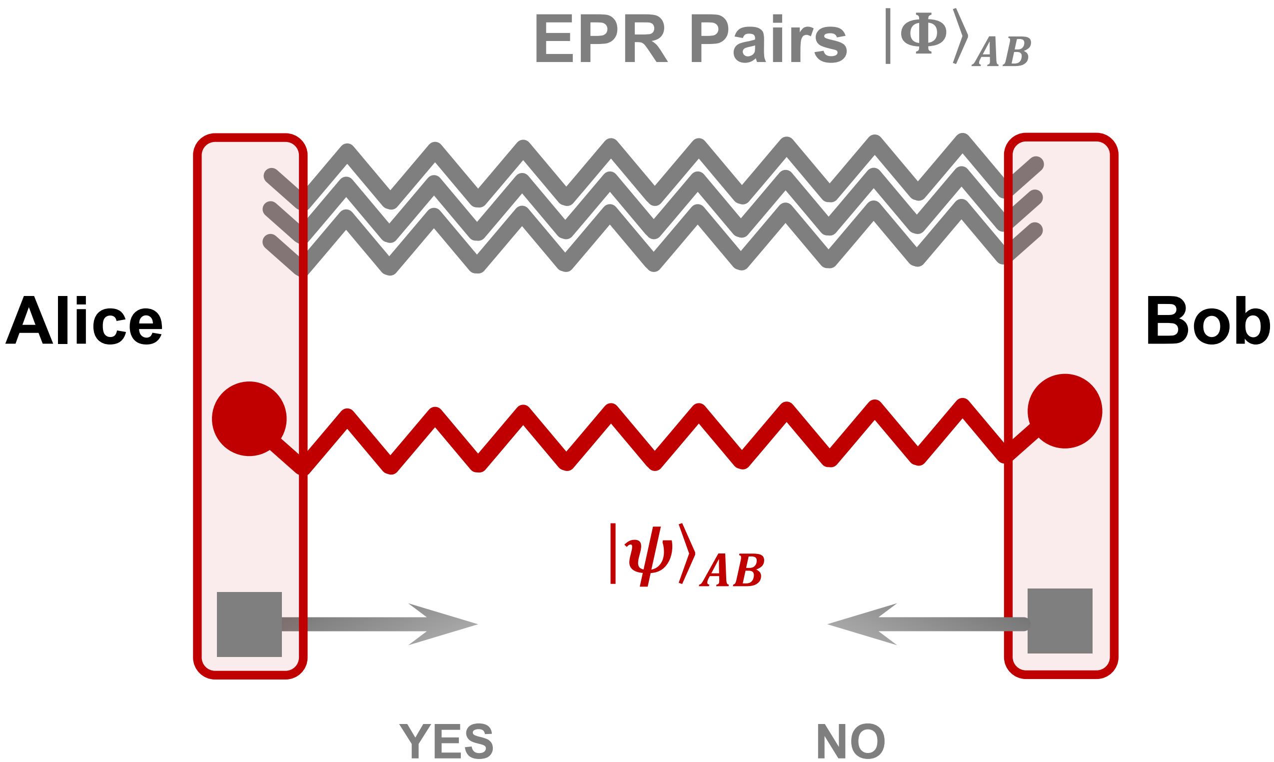


YES NO

$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

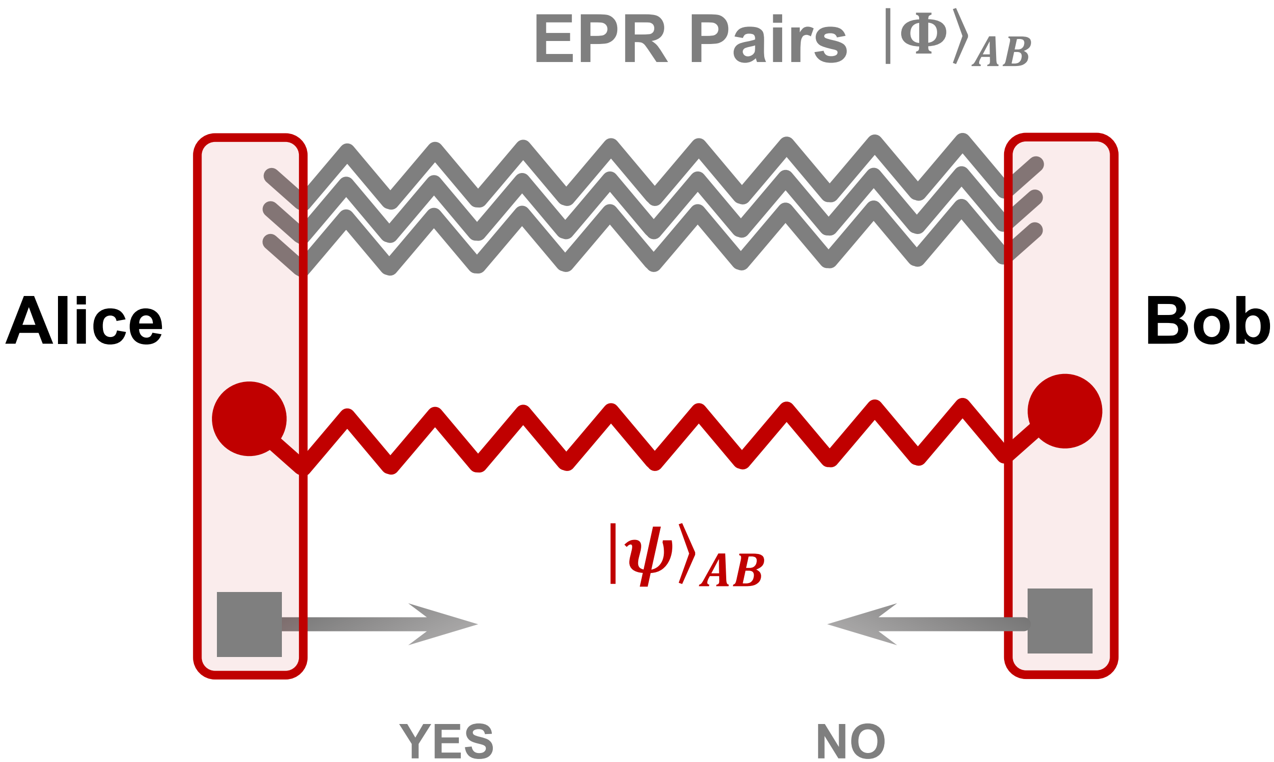
Testing Bipartite States



$$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$$

Two-Outcome Measurement

Testing Bipartite States

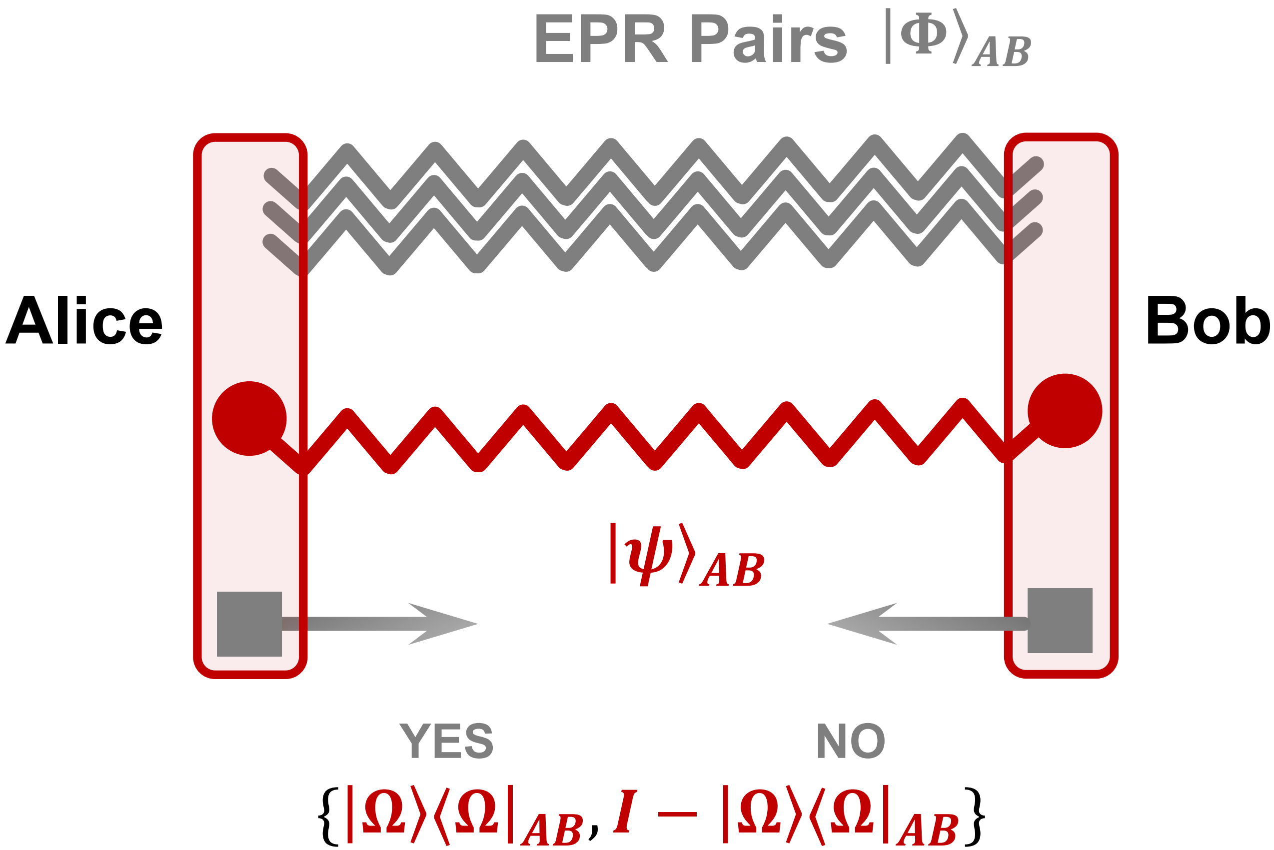


$\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$

Two-Outcome Measurement

$C(\Omega_{\mathrm{AB}})$ = **Minimum # of exchanged qubits to perform** $\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$

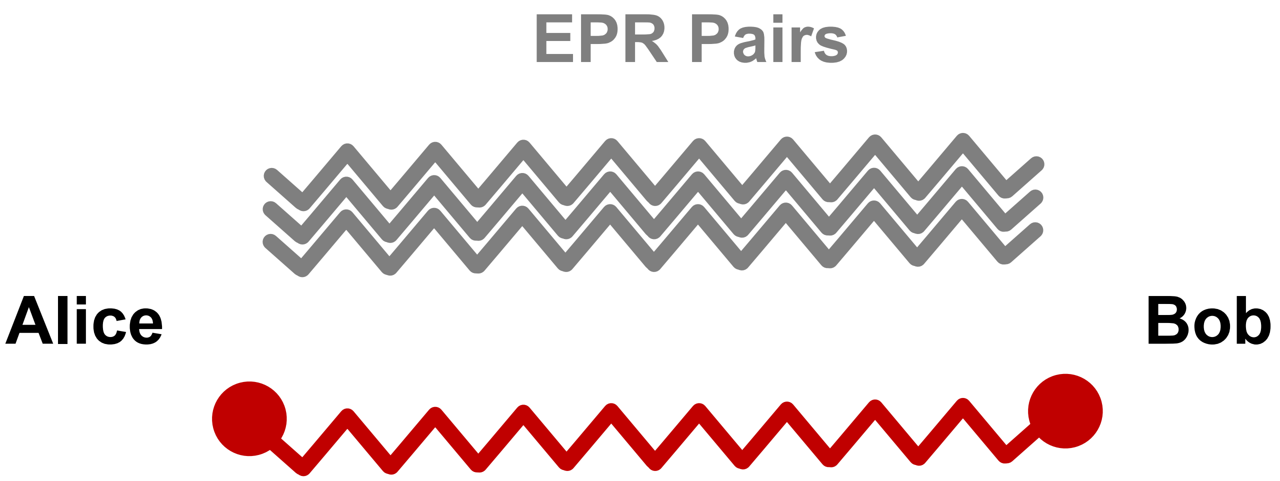
Testing Bipartite States



Two-Outcome Measurement

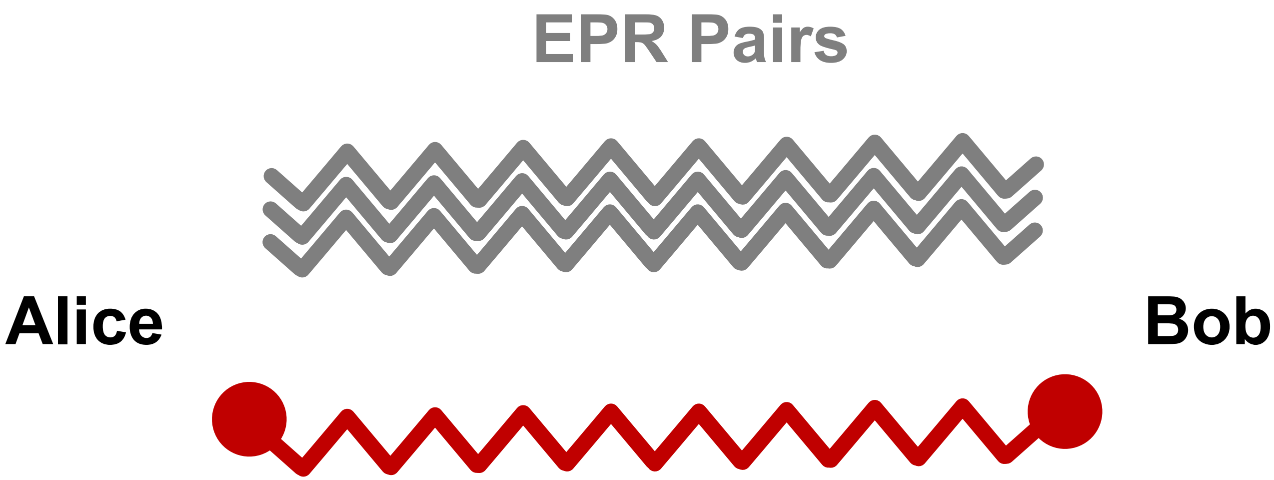
$C_\varepsilon(\Omega_{AB}) =$ **Minimum # of exchanged qubits to perform ε approximation of** $\{|\Omega\rangle\langle\Omega|_{AB}, I - |\Omega\rangle\langle\Omega|_{AB}\}$

Testing Bipartite States



What property of $|\Omega\rangle_{AB}$ determines $C_\varepsilon(\Omega_{AB})$?

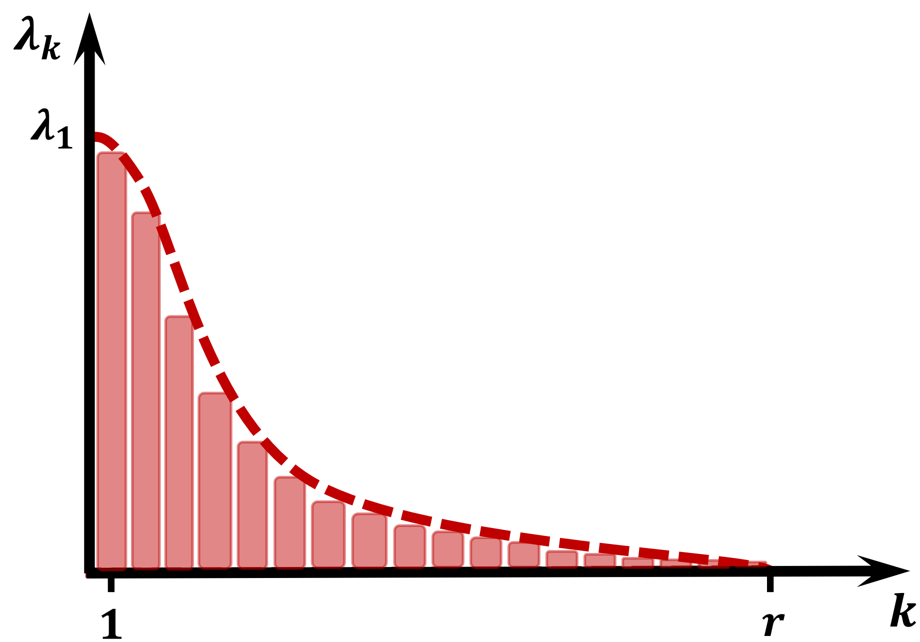
Testing Bipartite States



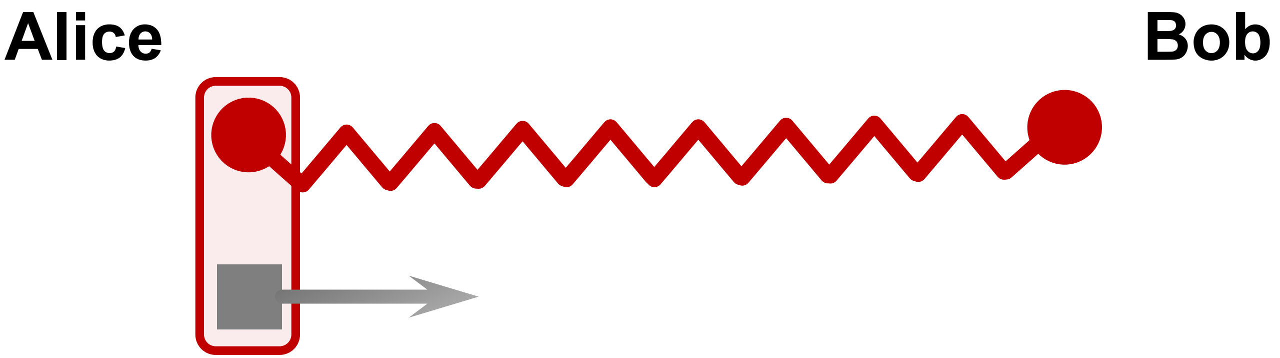
$$|\Omega\rangle_{AB} = \sum_{k=1}^{r} \sqrt{\lambda_k}\,|k\rangle_A |k\rangle_B$$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$$
$$\lambda_1 + \lambda_2 + \cdots + \lambda_r = 1$$

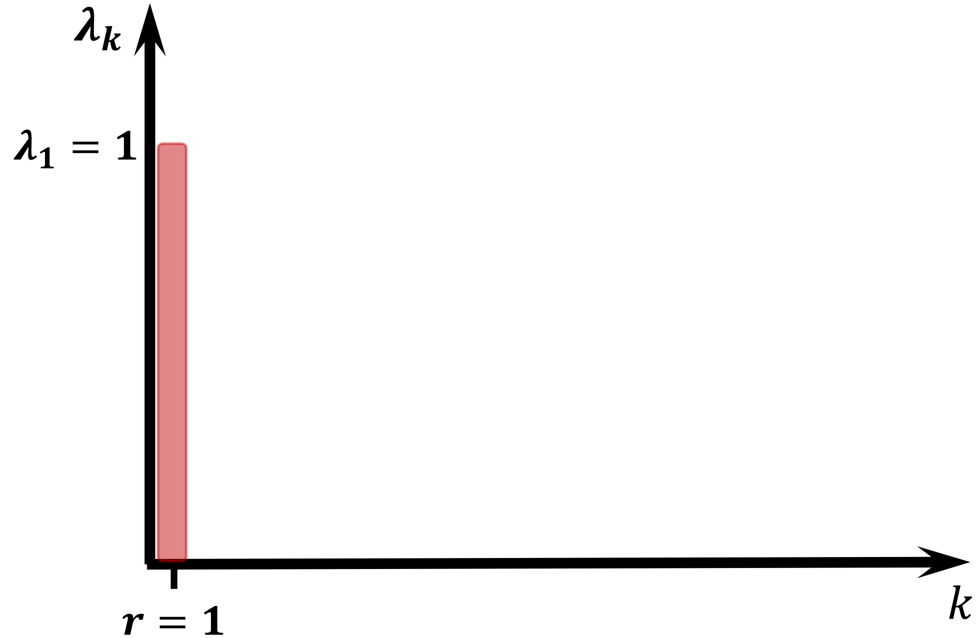
Schmidt Form



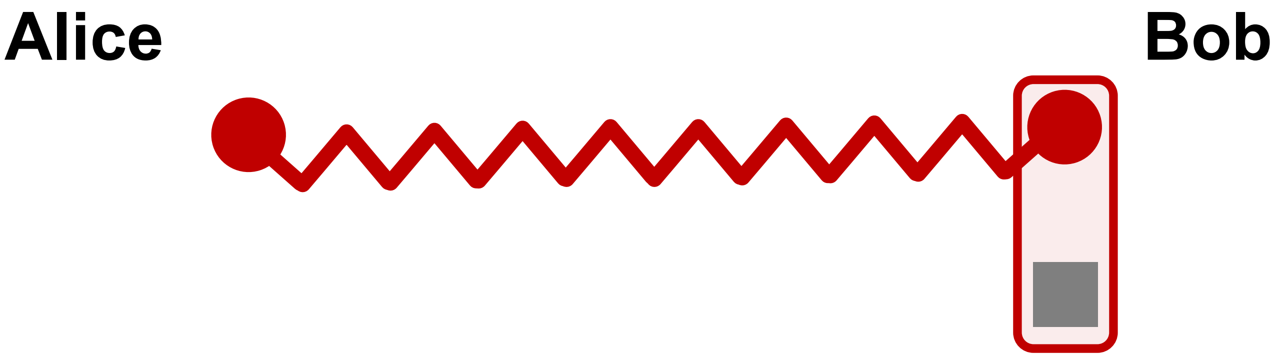
Testing Bipartite States



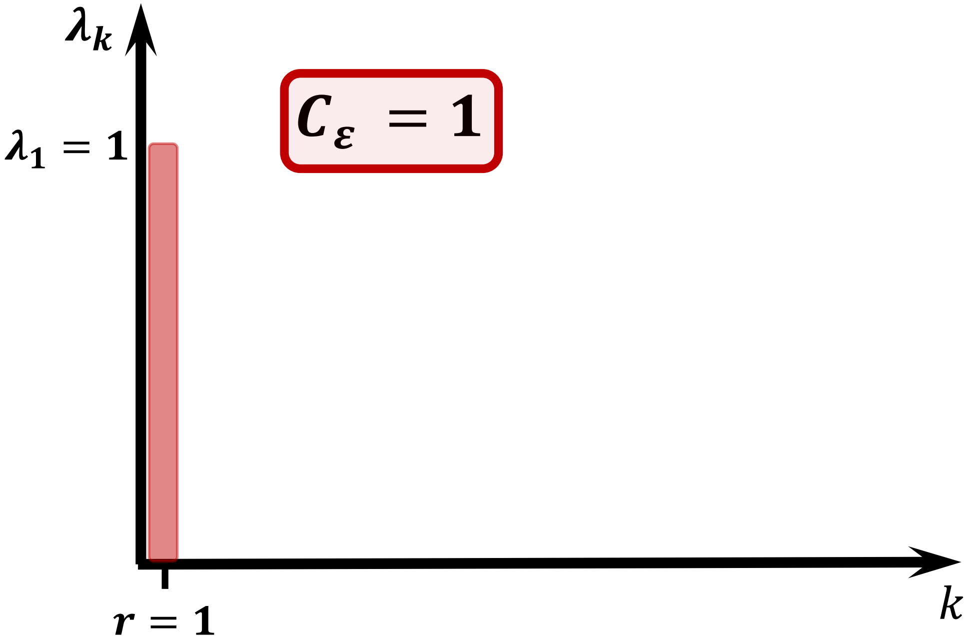
Testing $|0\rangle_A^{\otimes n}|0\rangle_B^{\otimes n}$



Testing Bipartite States



Testing $|0\rangle_A^{\otimes n}|0\rangle_B^{\otimes n}$

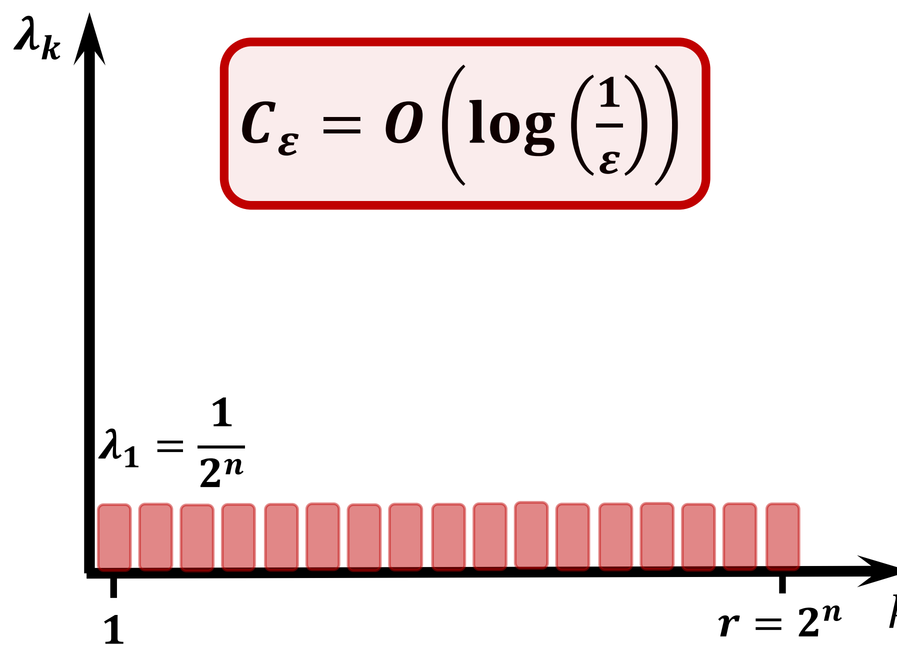


Testing Bipartite States

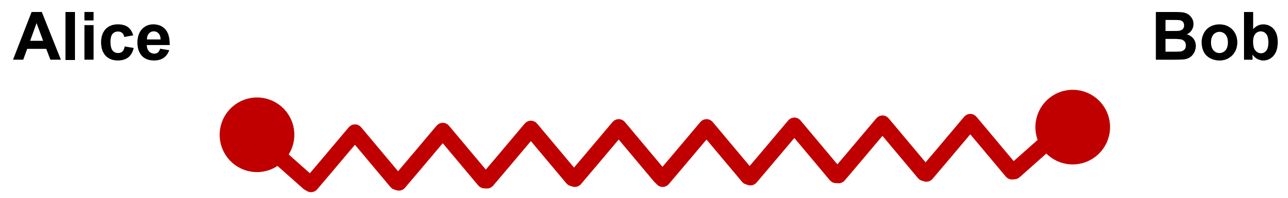
[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

$$C_\varepsilon = O\left(\log\left(\frac{1}{\varepsilon}\right)\right)$$



Testing Bipartite States

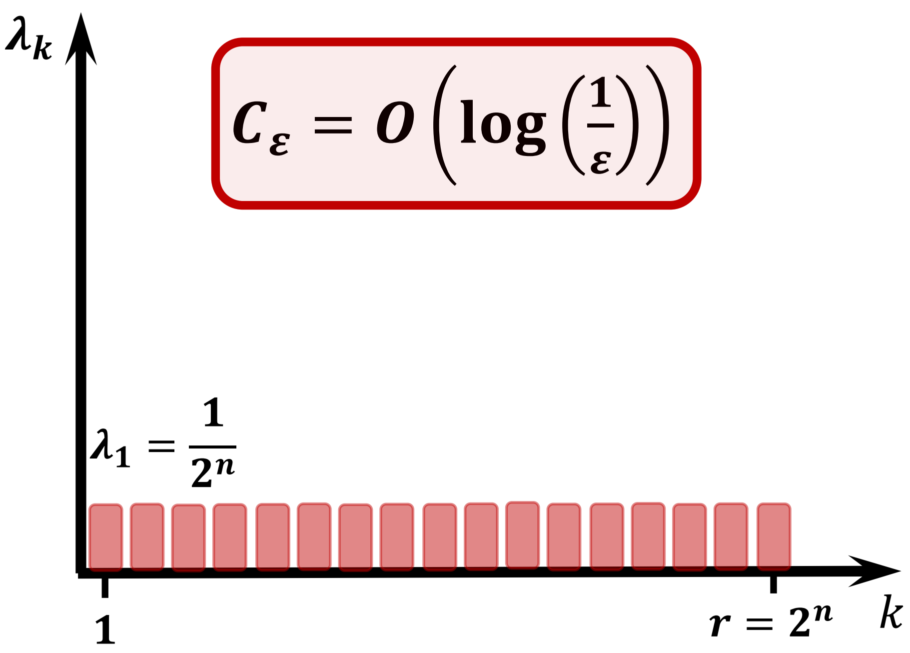


[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

$\{U_1, \ldots, U_d\}$ **with $d = 1/\varepsilon^c$ s.t.**



Testing Bipartite States

[AHL+14]

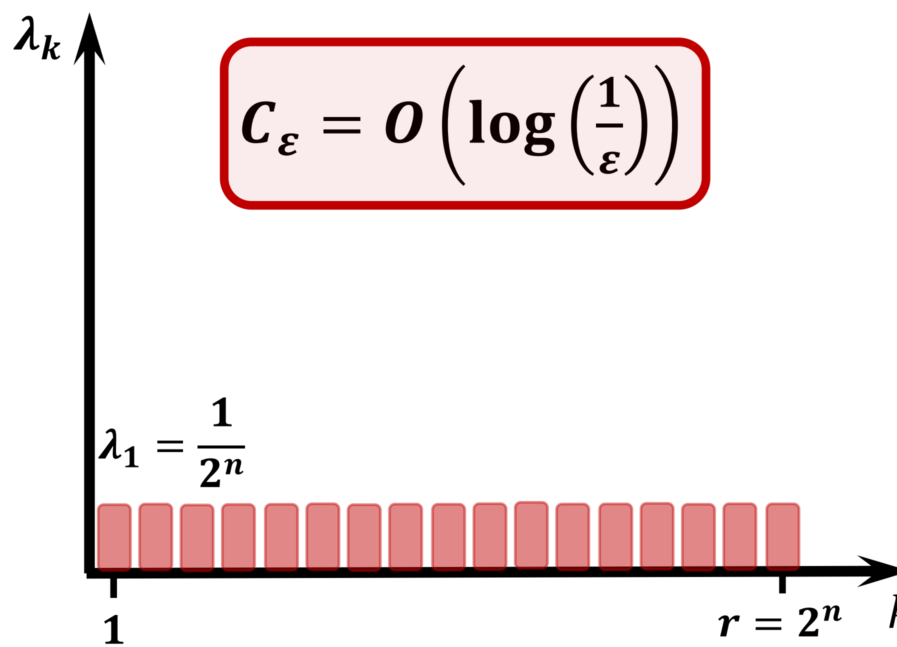
Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

$\{U_1, \dots, U_d\}$ **with $d = 1/\varepsilon^c$ s.t.**

$$\frac{1}{d}\sum_{k=1}^{d} U_k \otimes U_k^* \approx_\varepsilon |\mathrm{EPR}\rangle\langle \mathrm{EPR}|^{\otimes n}$$

$$C_\varepsilon = O\left(\log\left(\frac{1}{\varepsilon}\right)\right)$$



Testing Bipartite States

$$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}|k\rangle$$

[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

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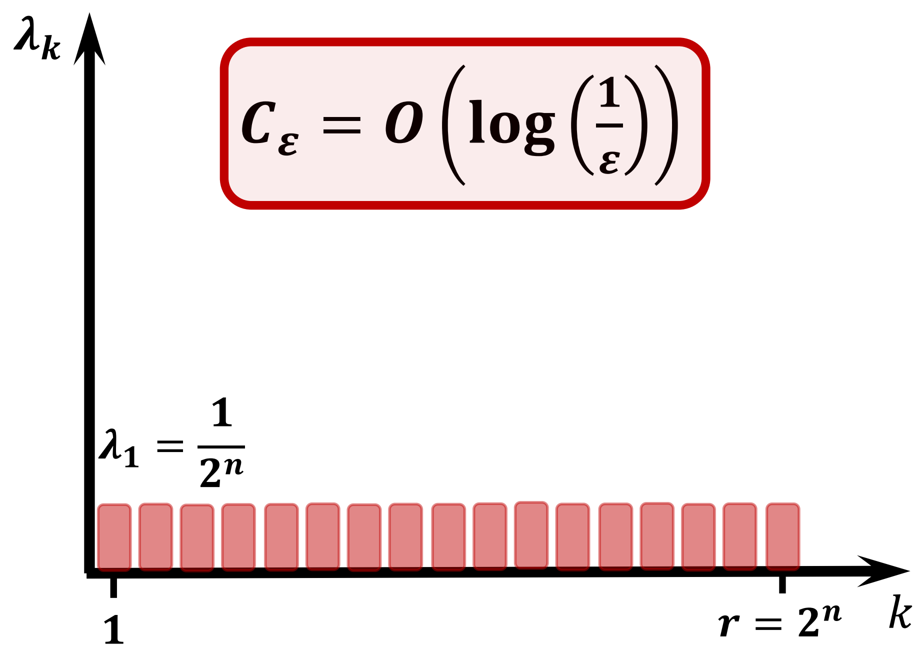

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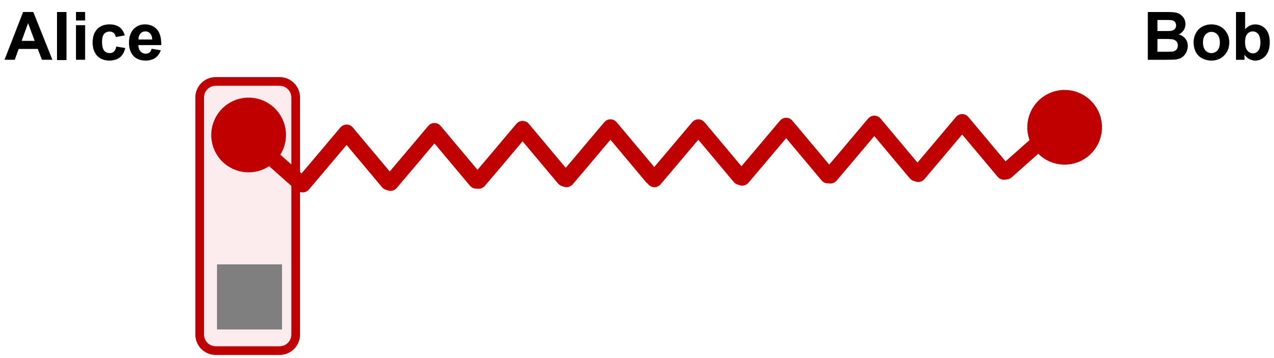
Testing Bipartite States



$$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}|k\rangle_{\blacksquare} \otimes (U_k \otimes I)|\psi\rangle_{AB}$$

[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

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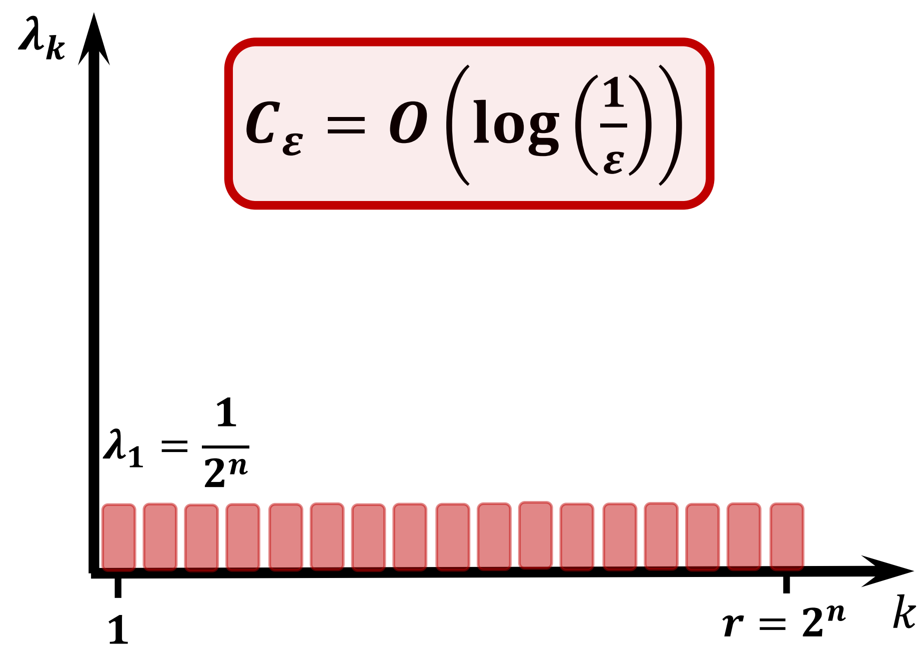
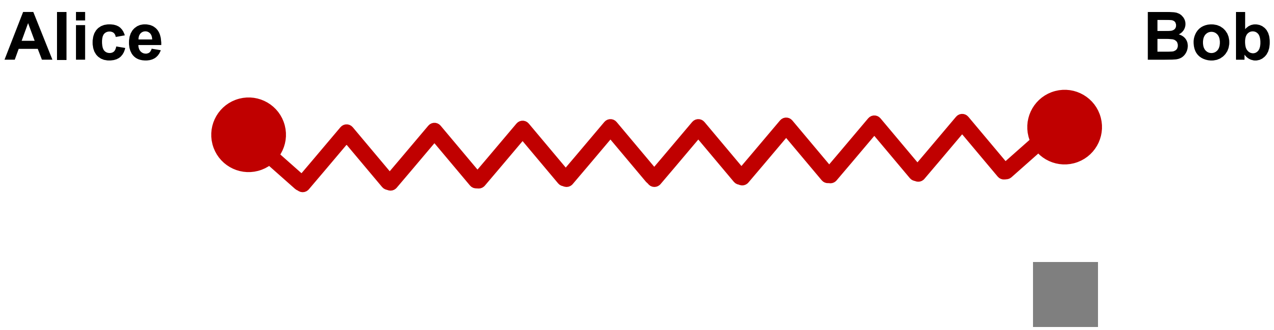


Testing Bipartite States



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[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

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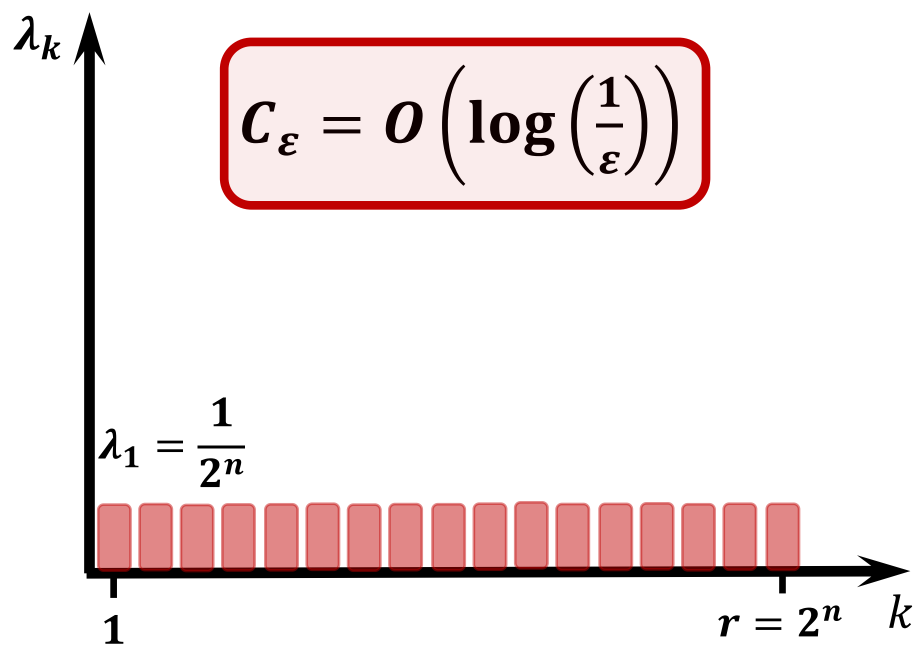
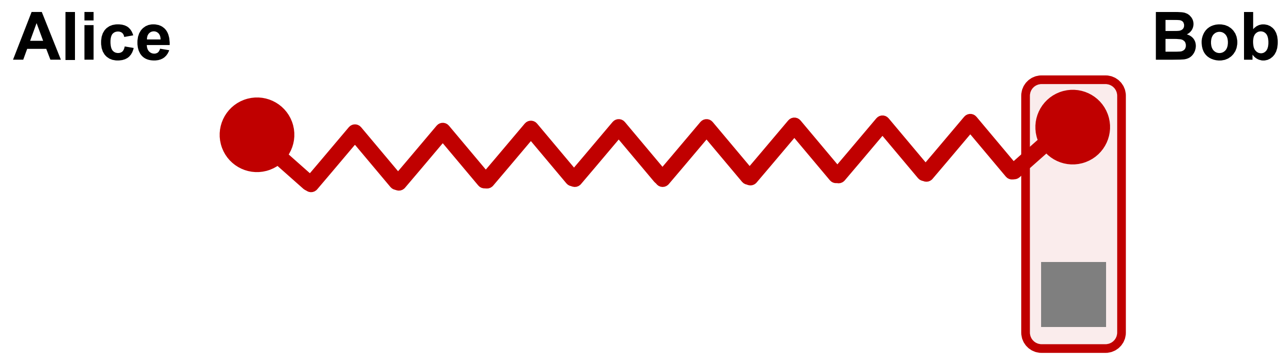


Testing Bipartite States



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[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

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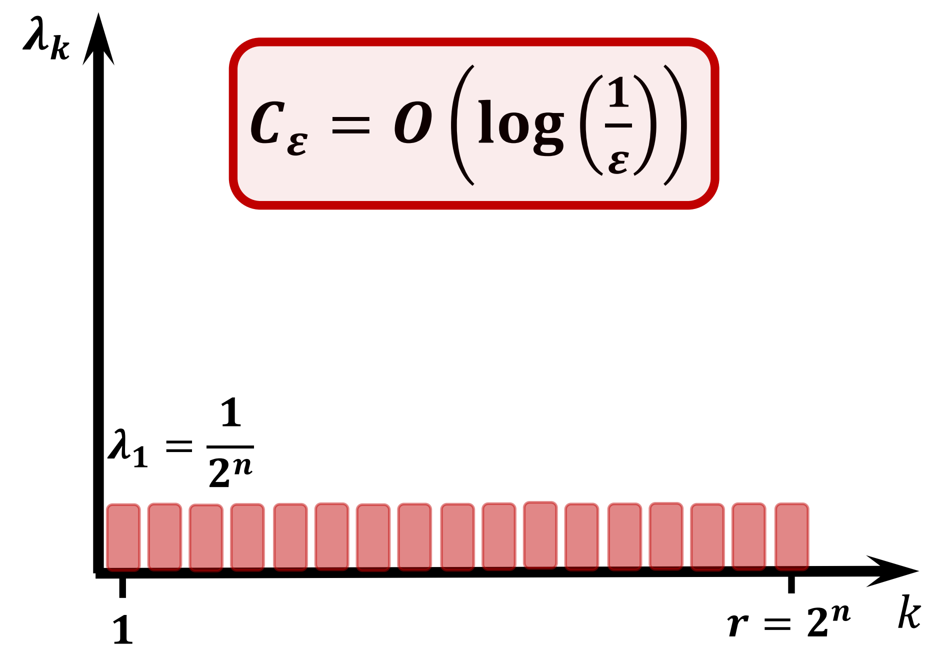
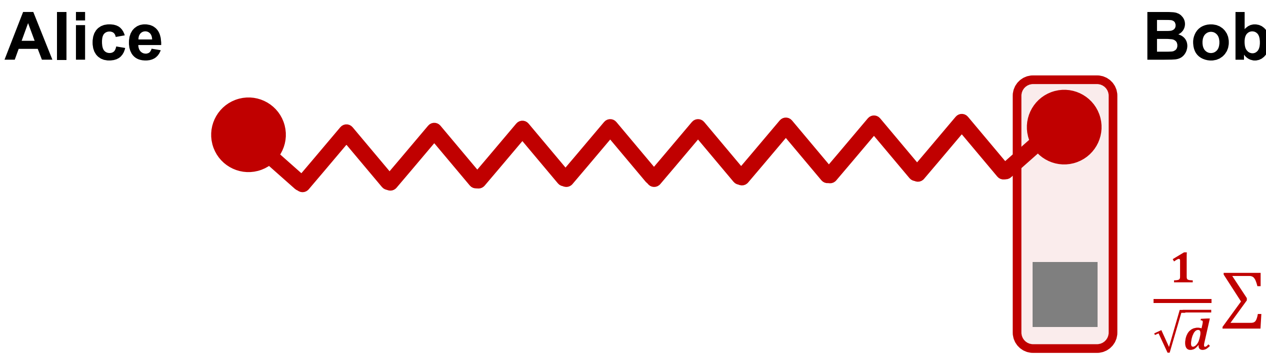


Testing Bipartite States



$$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}\langle k|_{\blacksquare}$$

$$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}|k\rangle_{\blacksquare} \otimes (U_k \otimes U_k^*)|\psi\rangle_{AB}$$

[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

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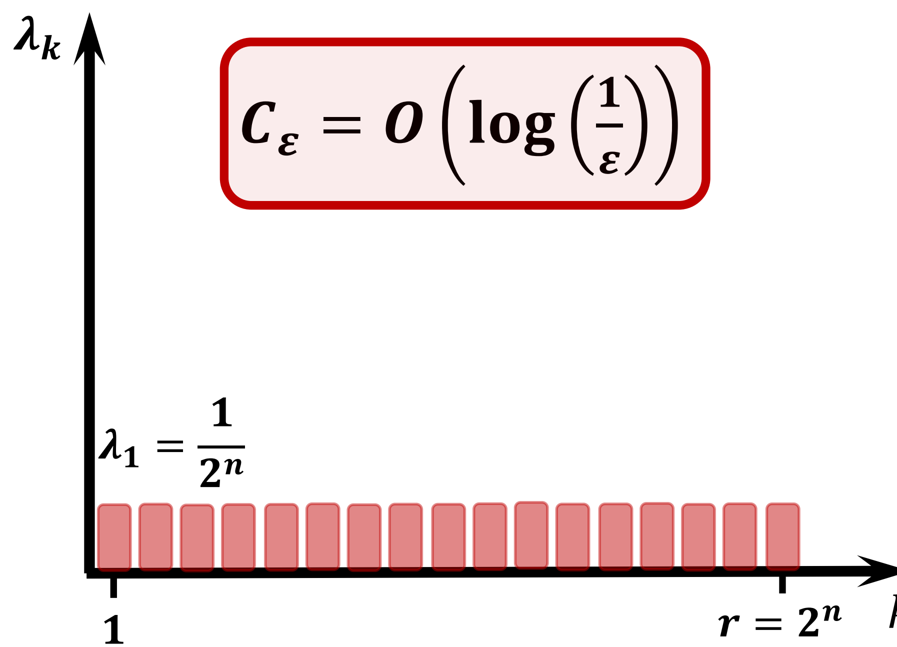
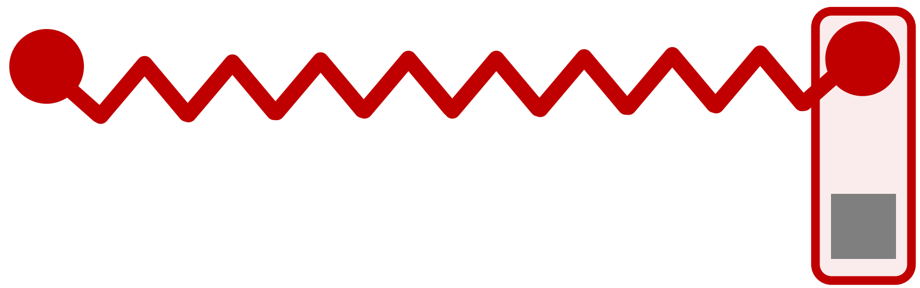


$$C_\varepsilon = O\left(\log\left(\frac{1}{\varepsilon}\right)\right)$$

Testing Bipartite States

Alice Bob



$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}\langle k|_{\blacksquare}$

$$\frac{1}{d}\sum_{k=1}^{d}(U_k \otimes U_k^*)\,|\psi\rangle_{AB}$$

[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

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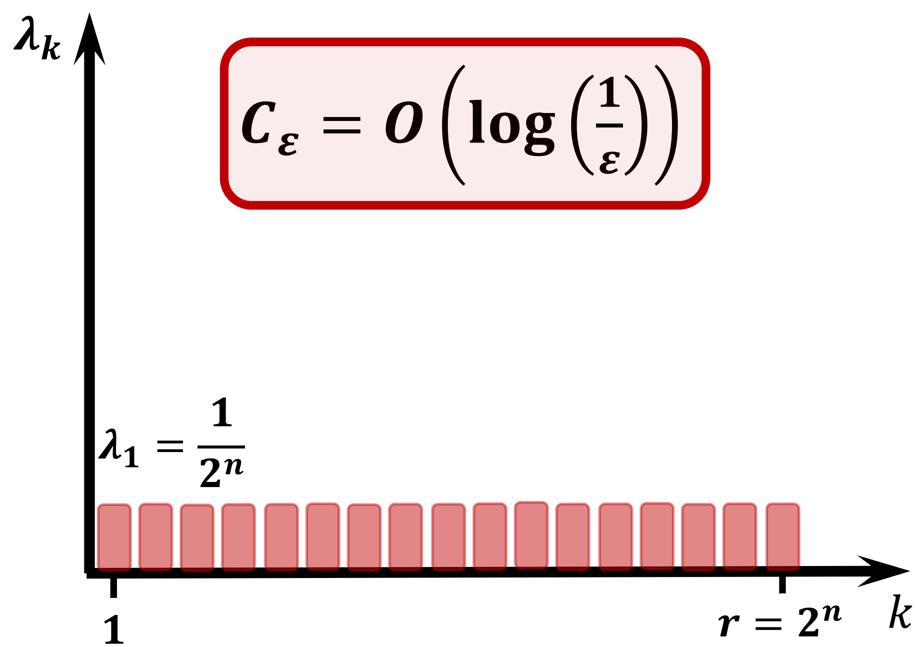
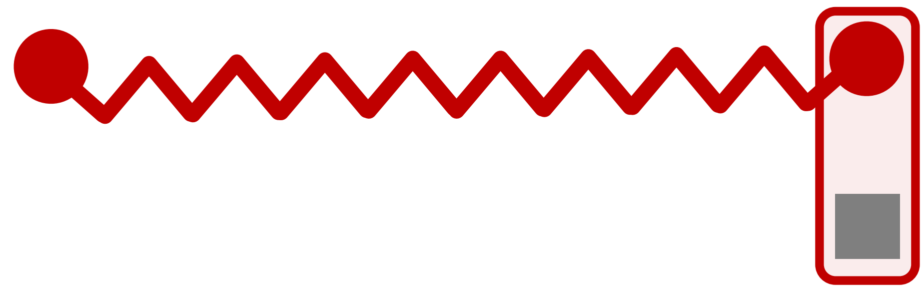


Testing Bipartite States



$$\frac{1}{\sqrt{d}}\sum_{k=1}^{d}\langle k|_{\blacksquare}$$

$$\frac{1}{d}\sum_{k=1}^{d}(U_k \otimes U_k^*)\,|\psi\rangle_{AB}$$

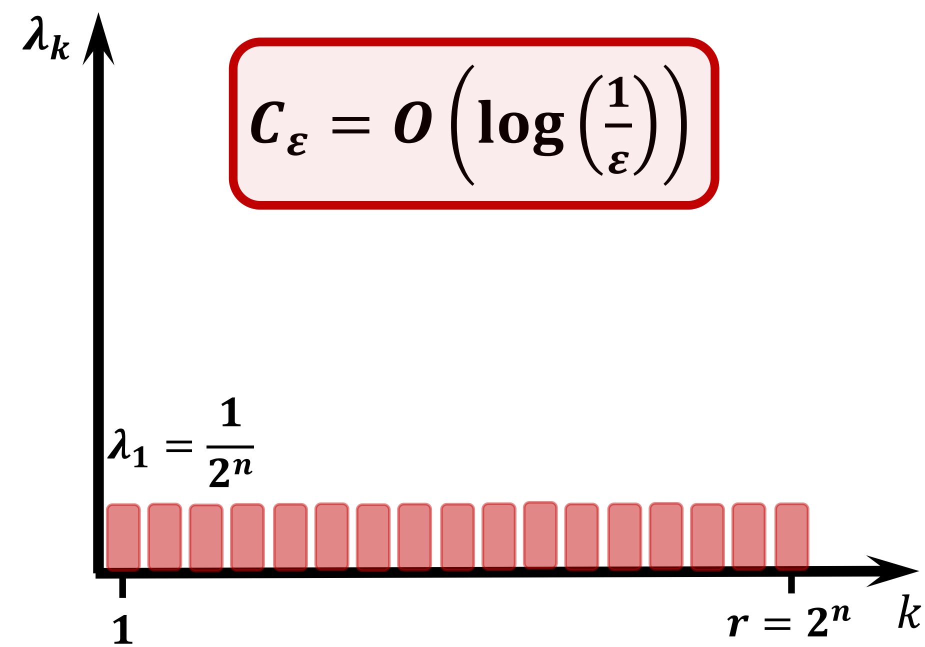
[AHL+14]

Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

$\{U_1, \ldots, U_d\}$ **with $d = 1/\varepsilon^c$ s.t.**

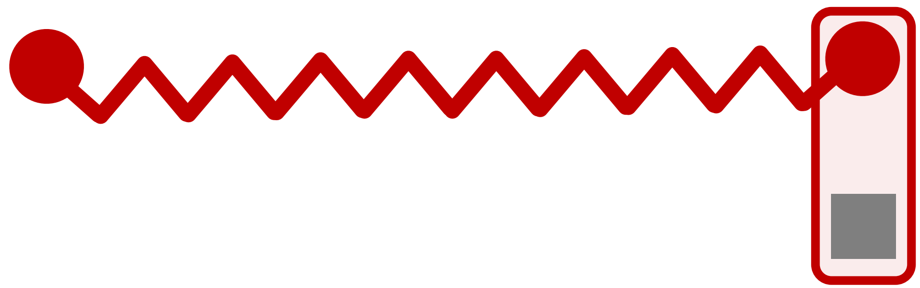
$$\frac{1}{d}\sum_{k=1}^{d} U_k \otimes U_k^* \approx_\varepsilon |\mathrm{EPR}\rangle\langle\mathrm{EPR}|^{\otimes n}$$



$$C_\varepsilon = O\left(\log\left(\frac{1}{\varepsilon}\right)\right)$$

Testing Bipartite States

Alice Bob



$\frac{1}{\sqrt{d}} \sum_{k=1}^{d} \langle k|_{\blacksquare}$

$$\approx_{\varepsilon} |\mathrm{EPR}\rangle\langle\mathrm{EPR}|^{\otimes n} \cdot |\psi\rangle_{AB}$$

[AHL+14]

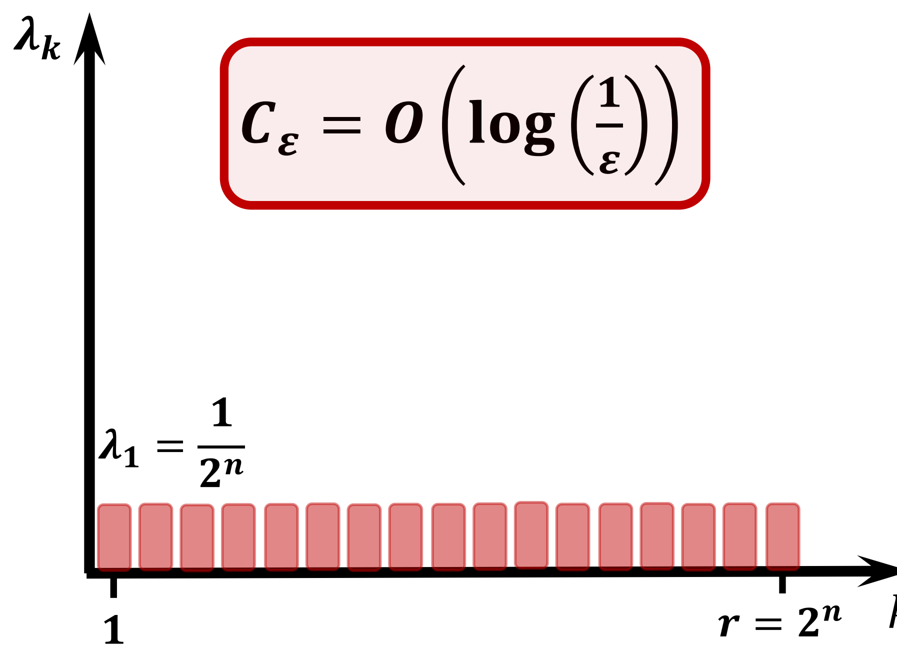
Testing n EPR pairs $|\mathrm{EPR}\rangle_{AB}^{\otimes n}$

Quantum Expanders

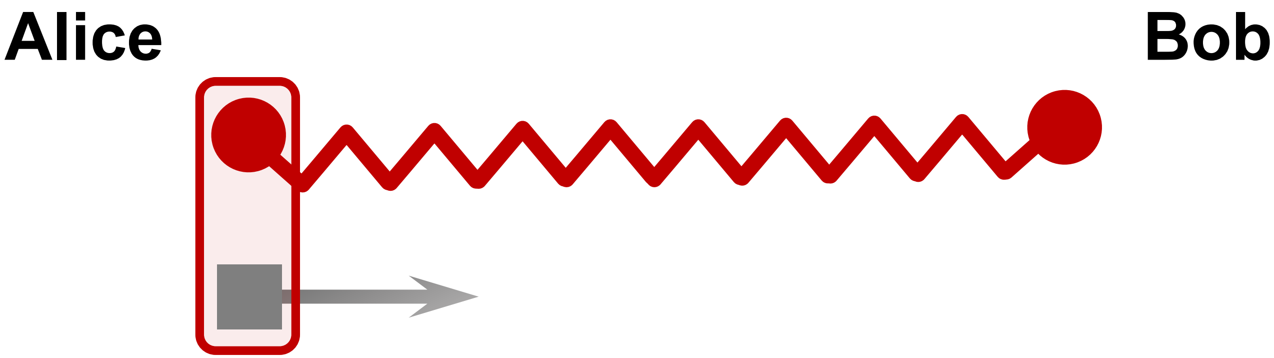
$\{U_1, \ldots, U_d\}$ **with $d = 1/\varepsilon^c$ s.t.**

$$\frac{1}{d} \sum_{k=1}^{d} U_k \otimes U_k^* \approx_{\varepsilon} |\mathrm{EPR}\rangle\langle\mathrm{EPR}|^{\otimes n}$$

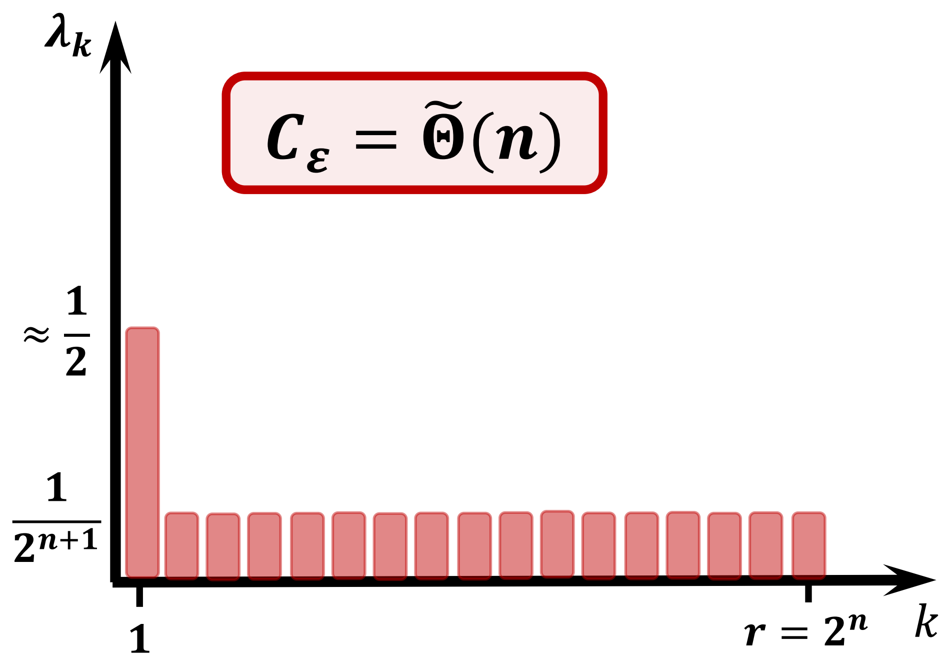
$$C_{\varepsilon} = O\left(\log\left(\frac{1}{\varepsilon}\right)\right)$$



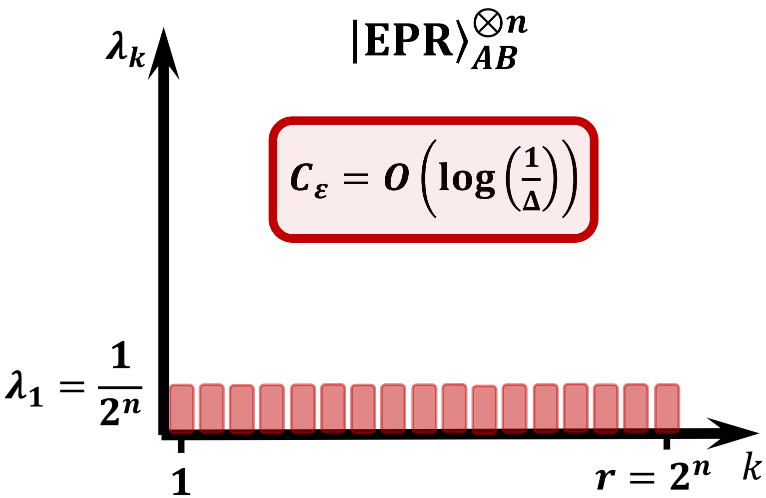
Testing Bipartite States

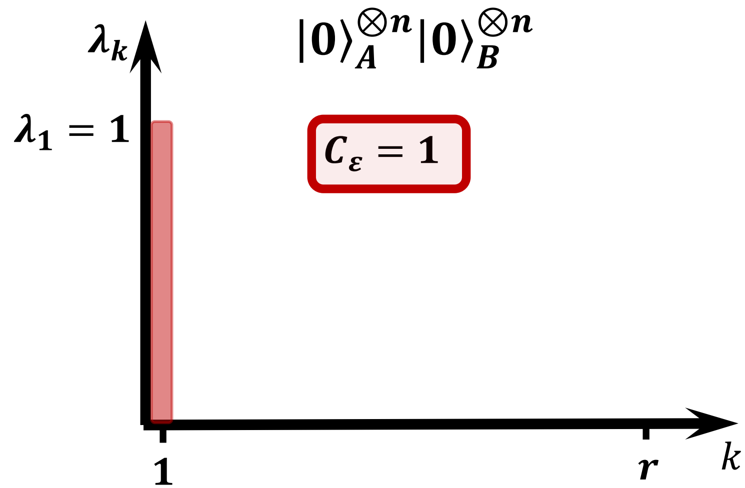


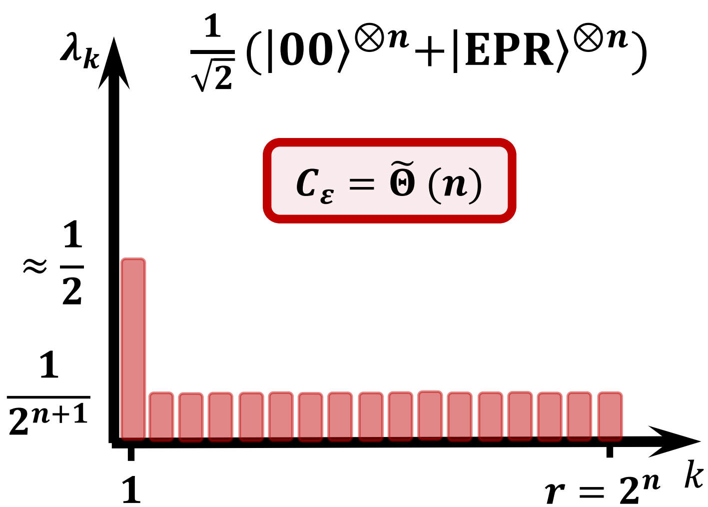
$$|\Omega\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle^{\otimes n} + |\mathrm{EPR}\rangle^{\otimes n}\right)$$



Recap

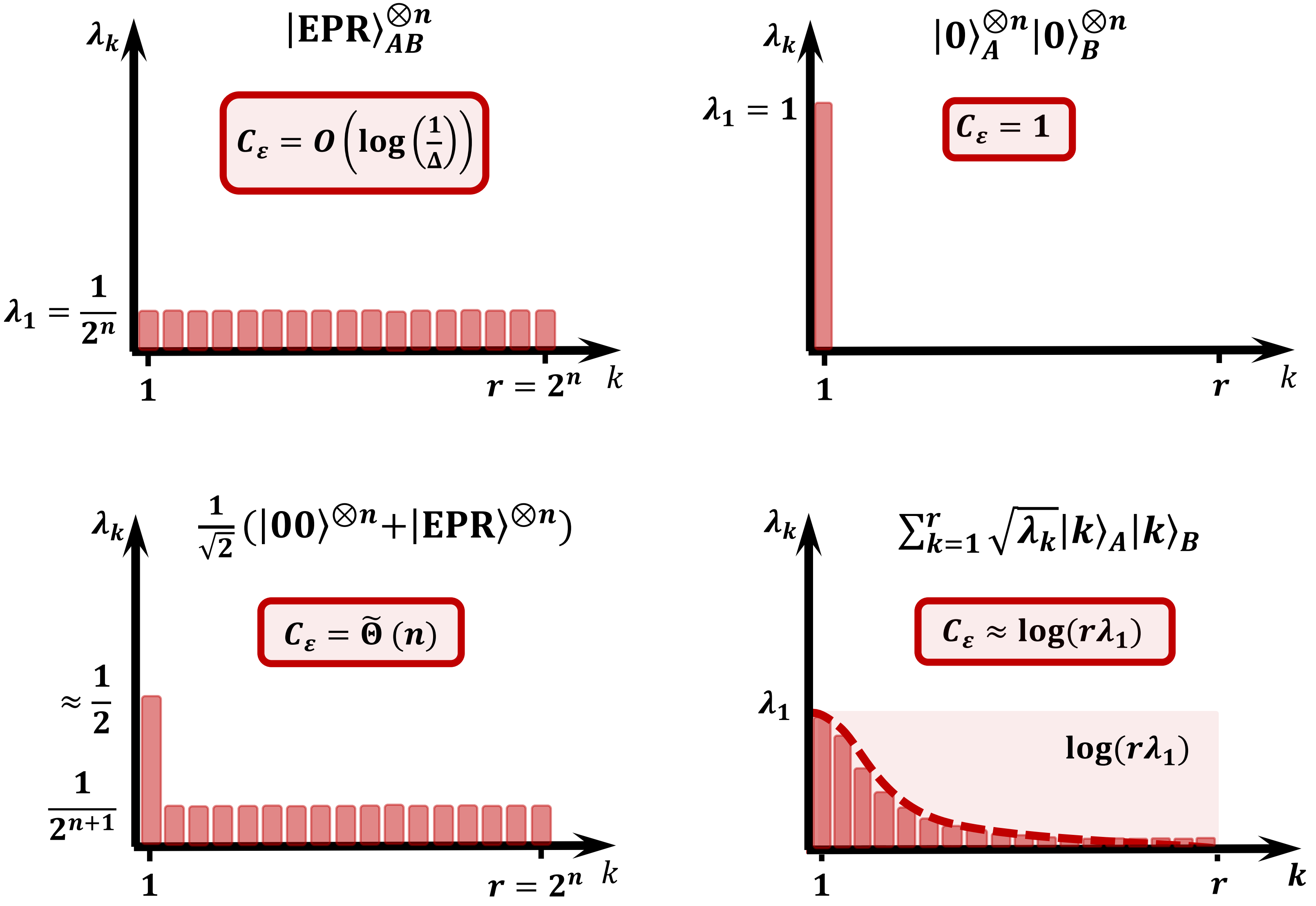






How spread out or concentrated λ_k are

Recap


$|\mathrm{EPR}\rangle_{AB}^{\otimes n}$
$C_\varepsilon = O\left(\log\left(\frac{1}{\Delta}\right)\right)$
λ_k
$\lambda_1 = \frac{1}{2^n}$
1
$r = 2^n$
k
$|0\rangle_A^{\otimes n}|0\rangle_B^{\otimes n}$
$C_\varepsilon = 1$
λ_k
$\lambda_1 = 1$
1
r
k
$\frac{1}{\sqrt{2}}(|00\rangle^{\otimes n} + |\mathrm{EPR}\rangle^{\otimes n})$
$C_\varepsilon = \tilde{\Theta}(n)$
λ_k
$\approx \frac{1}{2}$
$\frac{1}{2^{n+1}}$
1
$r = 2^n$
k
$\sum_{k=1}^{r} \sqrt{\lambda_k}|k\rangle_A|k\rangle_B$
$C_\varepsilon \approx \log(r\lambda_1)$
λ_k
λ_1
$\log(r\lambda_1)$
1
r
k

Entanglement Spread

[HW03] $$\mathrm{ES}(\Omega_A) = \log(r\lambda_1) = \log(r) - \log(1/\lambda_1)$$

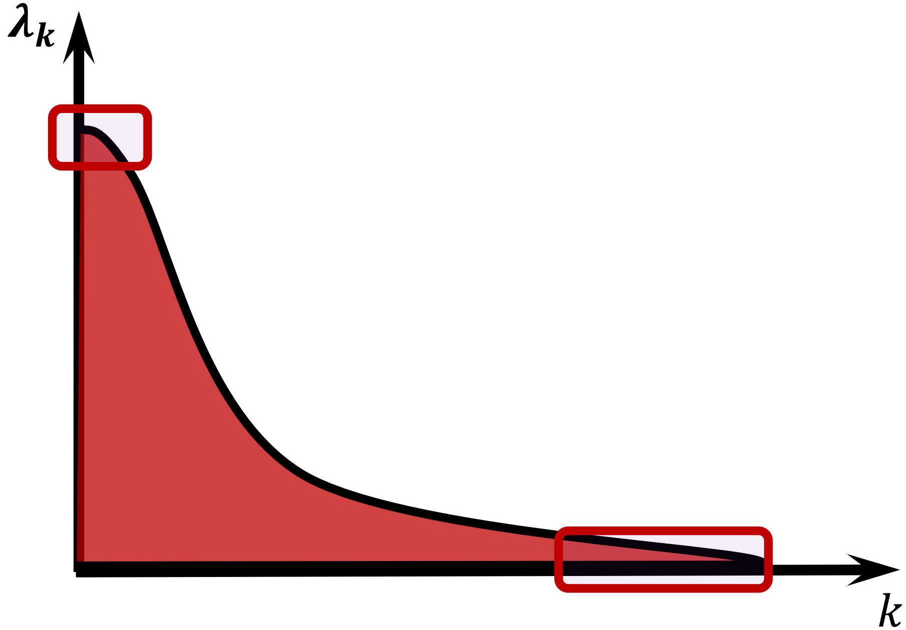
$$= S_{\max}(\Omega_A) - S_{\min}(\Omega_A)$$

$$\Omega_A = \mathrm{Tr}_B |\Omega\rangle\langle\Omega|_{AB}$$

ε –Smooth Entanglement Spread

$$\mathrm{ES}_\varepsilon(\Omega_A) = S^\varepsilon_{\max}(\Omega_A) - S^\varepsilon_{\min}(\Omega_A)$$

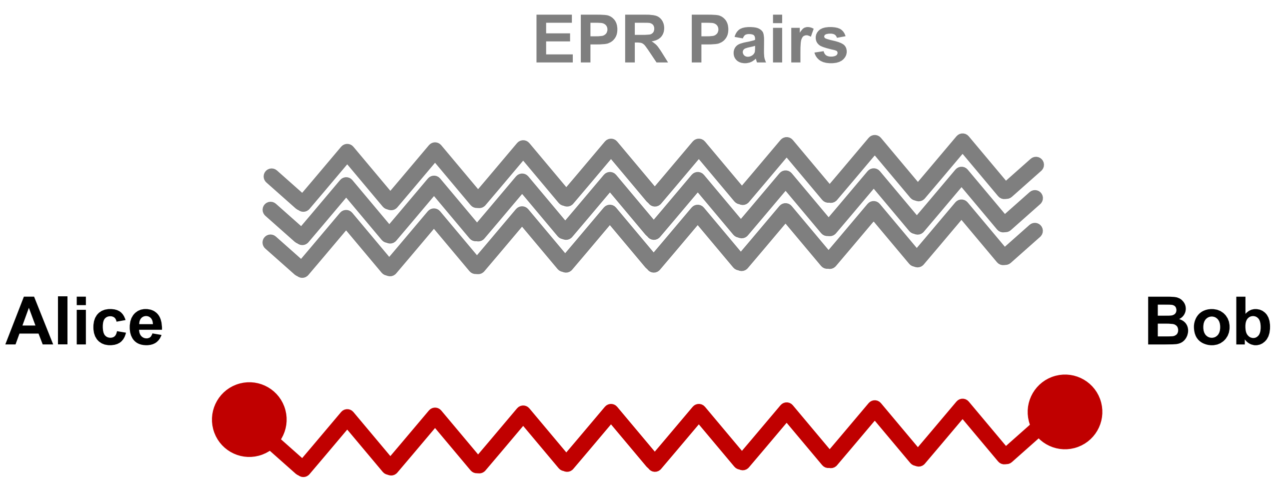
ε –Smooth min/max entropies



Communication Complexity ≥ Entanglement Spread

$$C_\varepsilon(\Omega_{AB}) \geq \mathbf{ES}_\varepsilon(\Omega_A) = S^\varepsilon_{\max}(\Omega_A) - S^\varepsilon_{\min}(\Omega_A)$$

[HW03, CH19, HL11]



Communication Complexity ≥ Entanglement Spread

$$C_\varepsilon(\Omega_{AB}) \geq \mathbf{ES}_\varepsilon(\Omega_A) = S^\varepsilon_{\max}(\Omega_A) - S^\varepsilon_{\min}(\Omega_A)$$

[HW03, CH19, HL11]

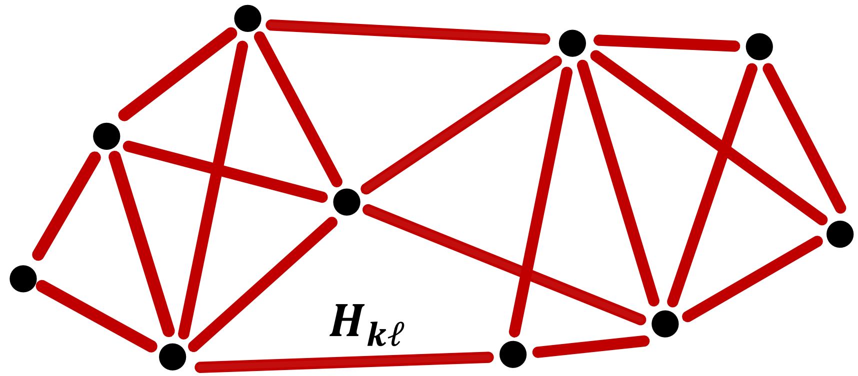
Holds even with EPR-assistance

Communication Complexity

Ground State Entanglement

Communication Complexity

Ground State Entanglement

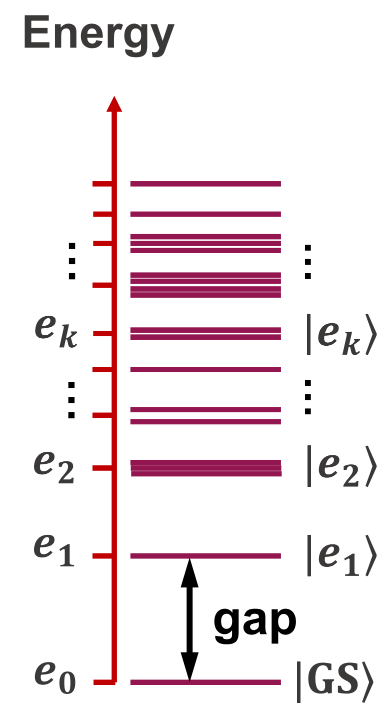


Local Hamiltonians $H = \sum_{k\sim\ell} H_{k\ell}$

Quantum Analog of CSP (e.g. Max-Cut)

$$H = \sum_{k\sim\ell}(e_{k\ell} Z_k Z_\ell + f_{kl} Y_k Y_\ell + g_{k\ell} X_k X_\ell)$$

(Hamiltonian need not be 2-local)



Ground State $|GS\rangle$

$e_0 = 0$

This Talk: Gapped Ground States

Gapped Ground States

- *Connected to central problems in physics (e.g. low T properties and novel phases of matter)*

- *Inherit locality of Hamiltonians*

$$(I - H/\|H\|)^c \approx |\mathbf{GS}\rangle\langle\mathbf{GS}|$$ AGSP Constructions [AKLV13]

- *Exhibit exponential decay of correlations*

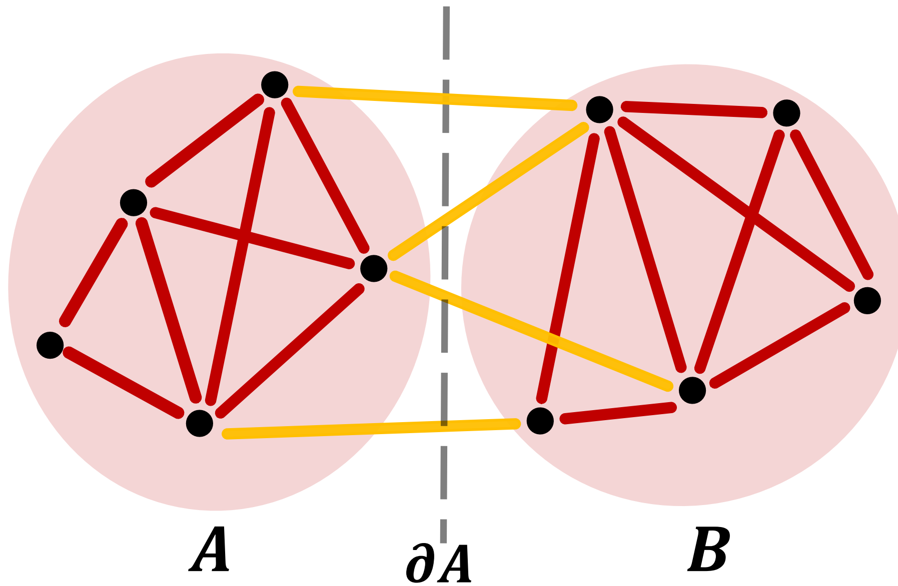
[Hastings04, HK05]

$$\langle A \rangle = \mathrm{Tr}[A \cdot \mathrm{GS}]$$

$$|\langle A \otimes B \rangle - \langle A \rangle \langle B \rangle| \leq \|A\| \cdot \|B\| \cdot e^{-\mathrm{dist}(A,B)/\xi}$$

- *Short-range entanglement*

Ground State Entanglement



$$|\mathrm{GS}\rangle_{AB} = \sum_k \sqrt{\lambda_k}\,|k\rangle_A |k\rangle_B$$

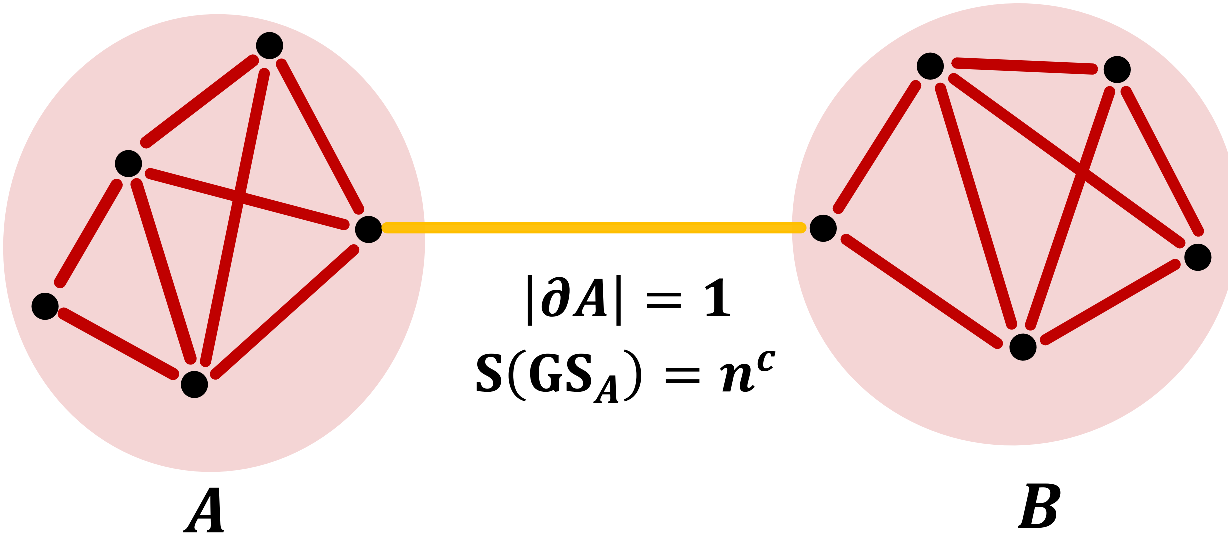
Entanglement Entropy $S(\mathrm{GS}_A) = -\sum_k \lambda_k \log(\lambda_k)$

[Hast07, ALV12, AKLV13] - **Area Law in 1D** $S(\mathrm{GS}_A) \le \widetilde{O}\left(\frac{|\partial A|}{\mathrm{gap}}\right)$

Used to find efficient MPS approximation

[AAG20, Abr19,…] - **Progress in 2D and Trees**

Ground State Entanglement



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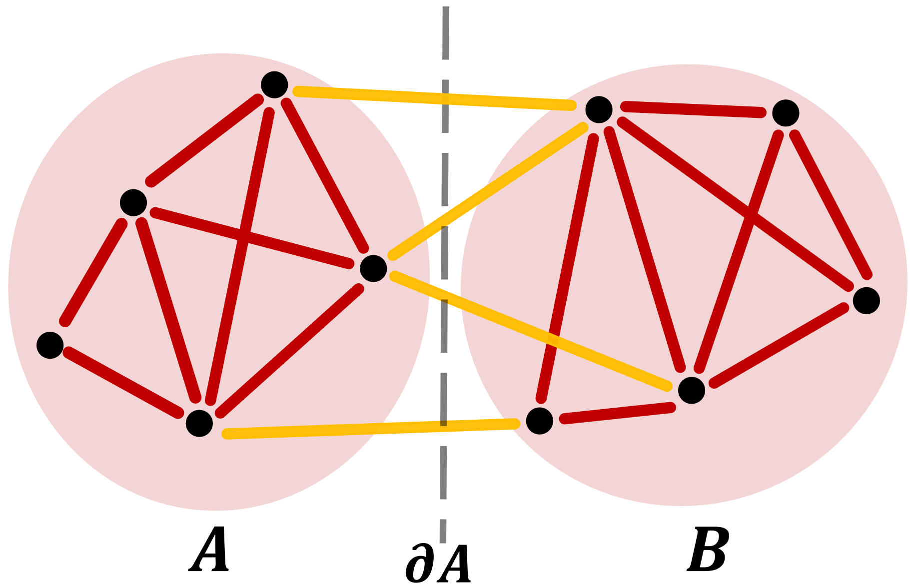
Used to find efficient MPS approximation

[AAG20, Abr19,…] - **Progress in 2D and Trees**

[AHL+14] - **Counter Example on General Graphs**

Ground State Entanglement



$$|\mathrm{GS}\rangle_{AB} = \sum_k \sqrt{\lambda_k}\,|k\rangle_A |k\rangle_B$$

Other structural properties for ground state entanglement?

$$\mathrm{ES}_\varepsilon(\mathrm{GS}_A) = \mathrm{S}^{\varepsilon}_{\max}(\mathrm{GS}_A) - \mathrm{S}^{\varepsilon}_{\min}(\mathrm{GS}_A)$$

Our Result: Area law for Entanglement Spread on *any* Graph

$$\mathrm{ES}_\varepsilon(\mathrm{GS}_A) \leq \widetilde{O}\left(\frac{|\partial A|}{\mathrm{gap}} \cdot \log\frac{1}{\varepsilon}\right)$$

← By designing a testing protocol

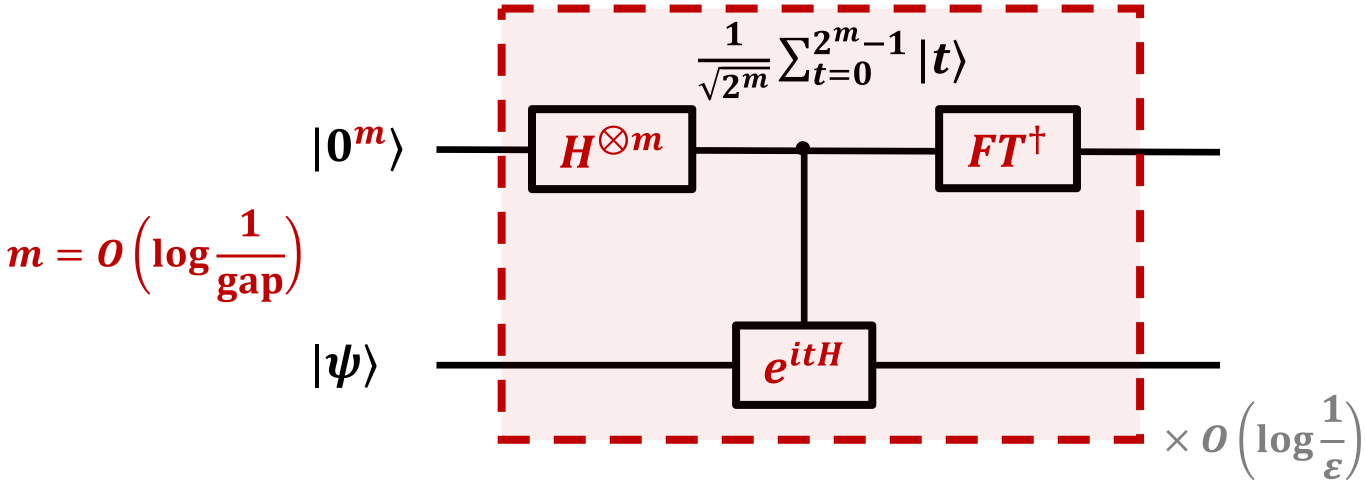
Communication Complexity ≥ Entanglement Spread

$$C_{\varepsilon}(\mathbf{GS}_{AB}) \geq \mathbf{ES}_{\varepsilon}(\mathbf{GS}_{A})$$

Testing Gapped Ground States

Measure energy $\langle\psi|H|\psi\rangle$
- **Yes**: $\langle\psi|H|\psi\rangle \le \mathbf{gap}/2$
- **No**: $\langle\psi|H|\psi\rangle > \mathbf{gap}/2$

Quantum Phase Estimation



$$W|0^m\rangle|\mathrm{GS}\rangle = |0^m\rangle|\mathrm{GS}\rangle$$

$$W|0^m\rangle|\Omega_k\rangle = \left(\sqrt{p_k}|0^m\rangle + \sqrt{1-p_k}|0^\perp\rangle\right)|\Omega_\mathrm{k}\rangle, \qquad p_k \ll 1$$

Repeat for $O\left(\log\frac{1}{\varepsilon}\right)$ to get $p_k = O(\varepsilon)$

Communication Complexity ≥ Entanglement Spread

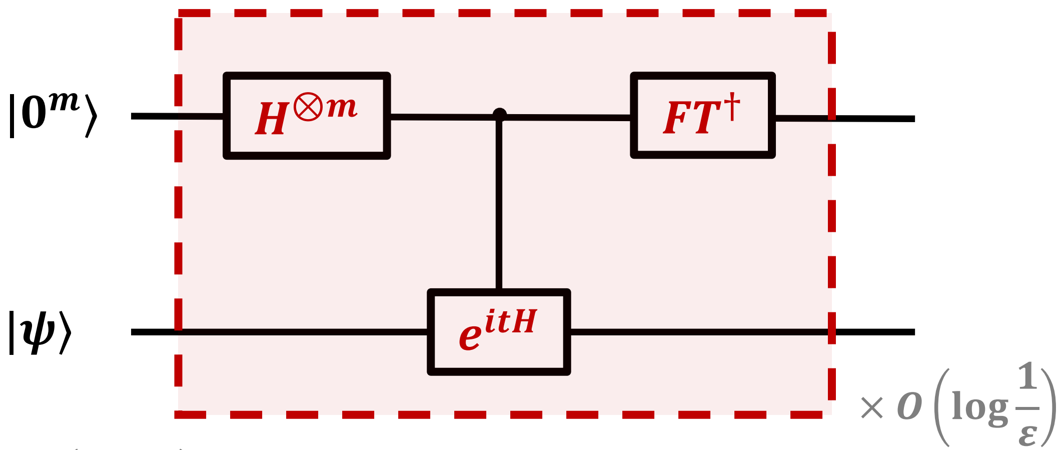
$$C_\varepsilon(\mathbf{GS}_{AB}) \geq \mathbf{ES}_\varepsilon(\mathbf{GS}_A)$$

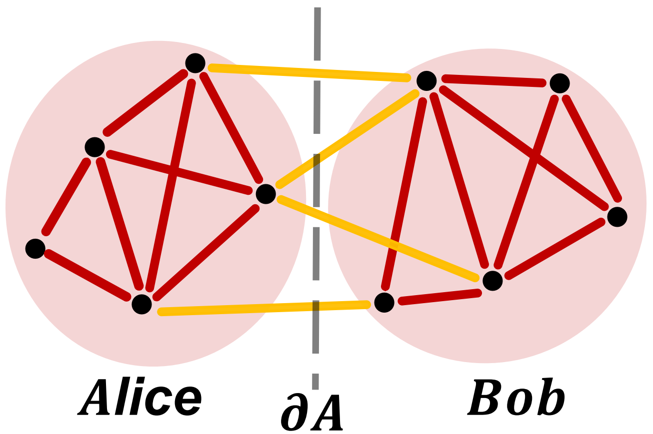
Area law for Entanglement Spread on *any* Graph

$$\mathbf{ES}_\varepsilon(\mathbf{GS}_A) \leq \widetilde{O}\left(\frac{|\partial A|}{\mathbf{gap}} \cdot \log\frac{1}{\varepsilon}\right)$$

Testing Gapped Ground States

Communication Protocol





For $O\left(\log\frac{1}{\varepsilon}\right)$ rounds

1) Alice shares $\frac{1}{\sqrt{1/\text{gap}}}\sum_{t=0}^{1/\text{gap}-1}|t\rangle_A|t\rangle_B$ $\quad O\left(\log\frac{1}{\text{gap}}\right)$ communications

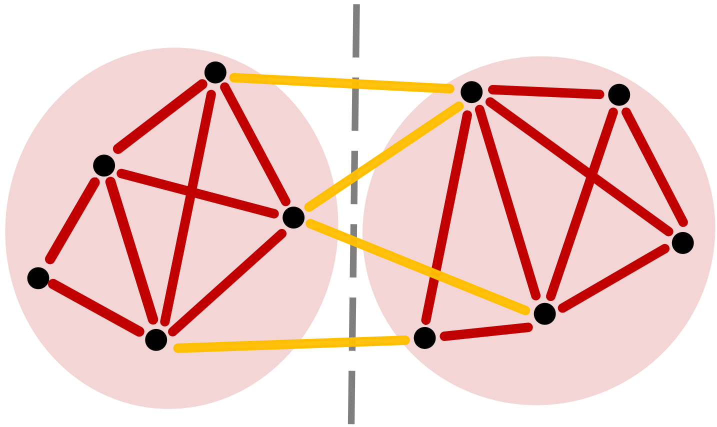
2) Jointly apply $e^{itH_{AB}}$ $\quad O\left(\frac{|\partial A|}{\text{gap}}\right)$ communications

3) Bob sends back $\quad O\left(\log\frac{1}{\text{gap}}\right)$ qubits

+

Overall Communication Cost: $\widetilde{O}(|\partial A|/\text{gap}\cdot\log 1/\varepsilon)$

Hamiltonian Simulation (Performing $e^{itH_{AB}}$)



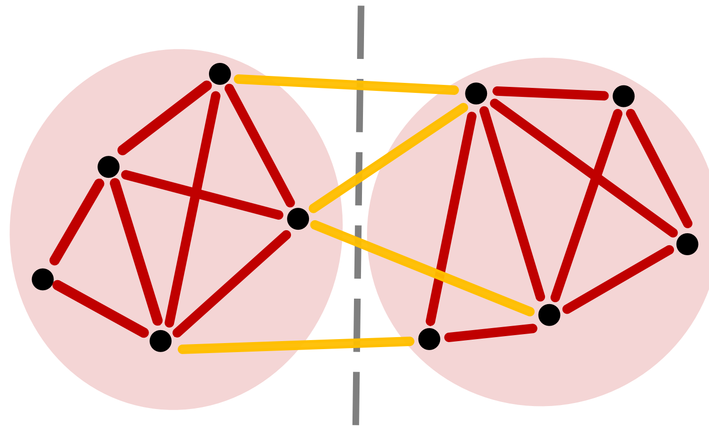
$$H_{AB} = H_A + H_{\partial A} + H_B$$

Depth of Hamiltonian simulation algorithms is $O(t\|H_{AB}\|)$

***Communication cost* of $e^{itH_{AB}}$ is $O(t\|H_{AB}\|)$**

How to improve this to $O(t\|H_{\partial A}\|)$?

Hamiltonian Simulation (Performing $e^{itH_{AB}}$)



$$H_{AB} = H_A + H_{\partial A} + H_B$$

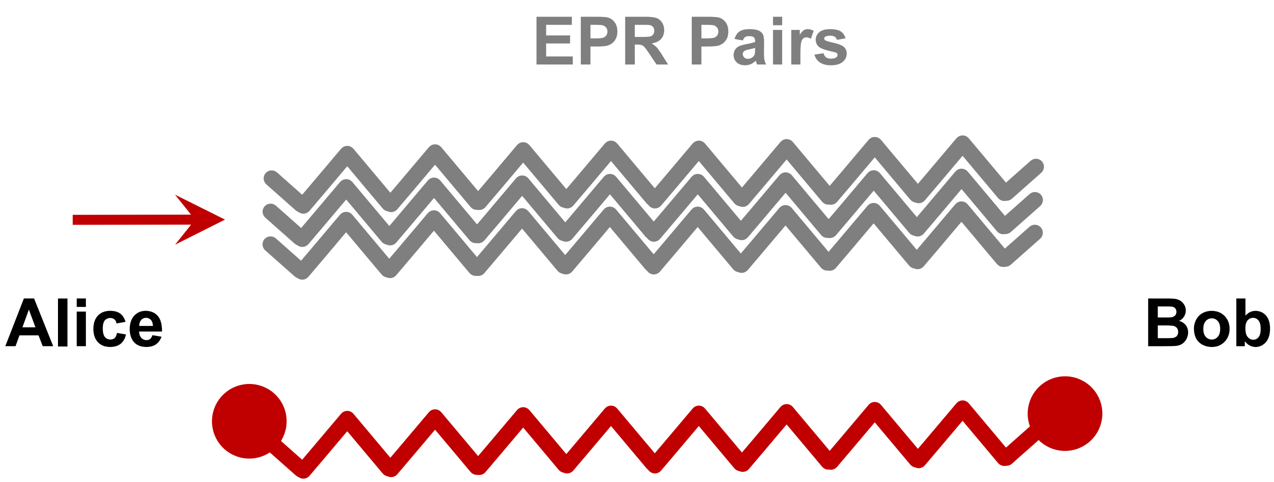
$e^{itH_{AB}} = e^{itH_A} \cdot e^{itH_B} \cdot e^{itH_{\partial A}}$ *when* $H_A, H_B, H_{\partial A}$ *Commute*

Interaction Picture: Time-dependent Hamiltonian [LW18]

$$H_I(t) = e^{-it(H_A+H_B)} \cdot H_{\partial A} \cdot e^{it(H_A+H_B)}$$

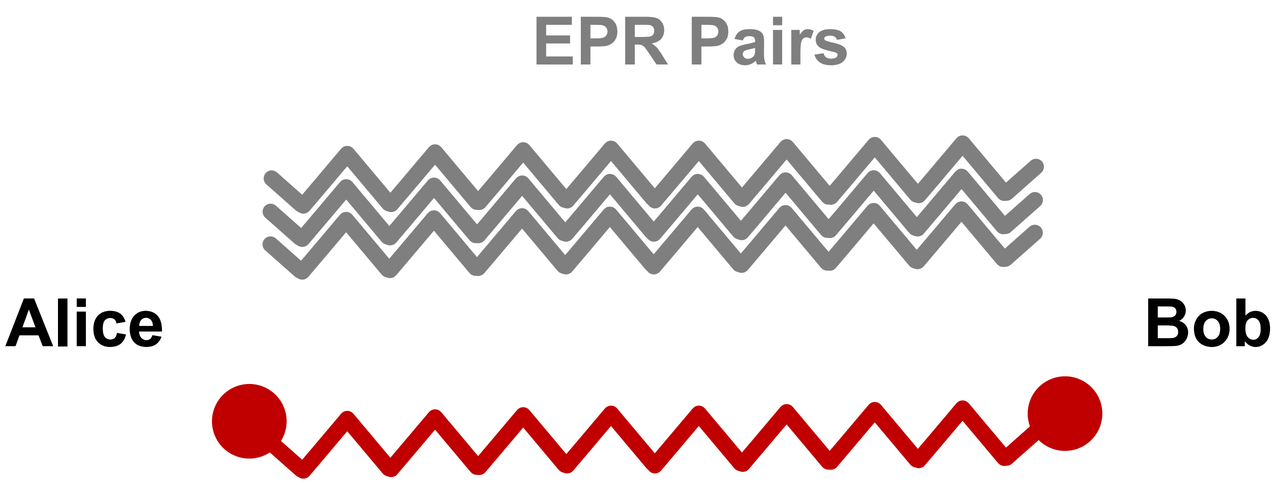
$$e^{itH_{AB}} = e^{itH_A} \cdot e^{itH_B} \cdot e^{\int_{\tau=0}^{t} iH_I(\tau)\,\mathrm{d}\tau}$$

Communication Cost of $O(t\|H_I\|) = O(t\|H_{\partial A}\|)$



Communication Complexity ≥ Entanglement Spread

$$C_\varepsilon(\mathrm{GS}_{AB}) \geq \mathrm{ES}_\varepsilon(\mathrm{GS}_A) = S^\varepsilon_{\max}(\mathrm{GS}_A) - S^\varepsilon_{\min}(\mathrm{GS}_A)$$



Communication Complexity ≥ Entanglement Spread

$$\boldsymbol{C_\varepsilon(\mathrm{GS}_{AB}) \geq \mathrm{ES}_\varepsilon(\mathrm{GS}_A) = S^\varepsilon_{\max}(\mathrm{GS}_A) - S^\varepsilon_{\min}(\mathrm{GS}_A)}$$

Time complexity of Alice and Bob doesn't matter so

Modify LCU [BCC+15] and use EPR-assistance to implement Taylor expansion of e^{iHt}

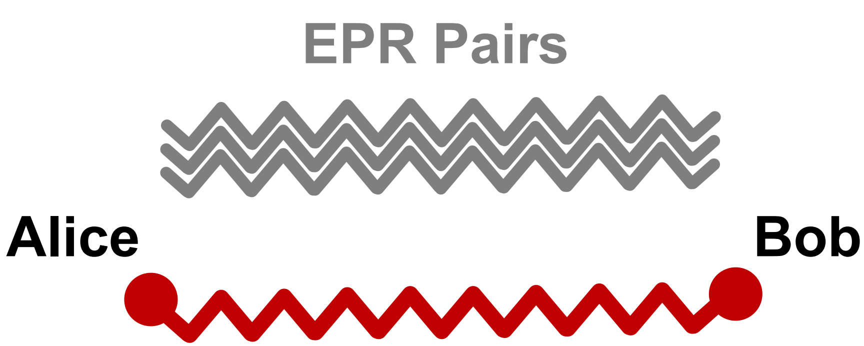
Summary

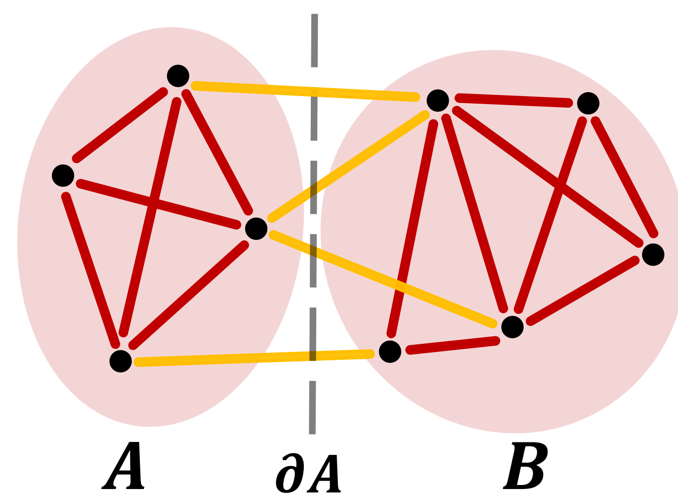
Communication Complexity ≥ Entanglement Spread

$$C_\varepsilon(\Omega_{AB}) \geq ES_\varepsilon(\Omega_A) = S^\varepsilon_{max}(\Omega_A) - S^\varepsilon_{min}(\Omega_A)$$

Area law for Entanglement Spread on *any* Graph

$$ES_\varepsilon(GS_A) \leq \widetilde{O}\left(\frac{|\partial A|}{gap} \cdot \log\frac{1}{\varepsilon}\right)$$





Improvement for Lattices

Sub-Area law for Entanglement Spread on *lattices* (Tight)

$$\mathrm{ES}_{\varepsilon}(\mathrm{GS}_A) \leq \widetilde{O}\left(\sqrt{\frac{|\partial A|}{\mathrm{gap}}} \cdot \log\frac{1}{\varepsilon}\right)$$

Gives evidence for Li-Haldane Conjecture [LH08] in physics

$$\mathrm{GS}_A \approx e^{-H_{\partial A}} \quad \textbf{Then} \quad \mathrm{ES}(\mathrm{GS}_A) = O\left(\sqrt{|\partial A|}\right)$$

Improvement for Lattices

Sub-Area law for Entanglement Spread on *lattices* (Tight)

$$\mathrm{ES}_{\varepsilon}(\mathrm{GS}_{A}) \leq \tilde{O}\left(\sqrt{\frac{|\partial A|}{\mathrm{gap}}} \cdot \log\frac{1}{\varepsilon}\right)$$

Implication for Entropy Area Law

Gapped ground states always have small Entanglement Spread

$$S_{\max}^{\varepsilon}(\mathrm{GS}_{\mathrm{A}}) - S_{\min}^{\varepsilon}(\mathrm{GS}_{\mathrm{A}})$$

$S_{\min}^{\varepsilon}(\mathrm{GS}_{\mathrm{A}})$ is **small** → **Entropy Area Law**

$S_{\min}^{\varepsilon}(\mathrm{GS}_{\mathrm{A}})$ is **large** → **Violated Entropy Area Law** [AHL+14]

Open questions

1) **Efficient** contraction of **tensor network** representation of **gapped** ground states from **entanglement spread** area law?

[AAJ16], [CPSV11]

2) Other applications for our **AGSP** based on **QPE** and **Hamiltonian simulation**?

3) Other **universal** properties of **gapped** ground states?

From Communication Complexity to an Entanglement Spread Area Law

Mehdi Soleimanifar (MIT)

(arxiv: 2004.15009)

Joint work with

Anurag Anshu (UC Berkeley)

Aram Harrow (MIT)