Certifying almost all quantum states with few single-qubit measurements

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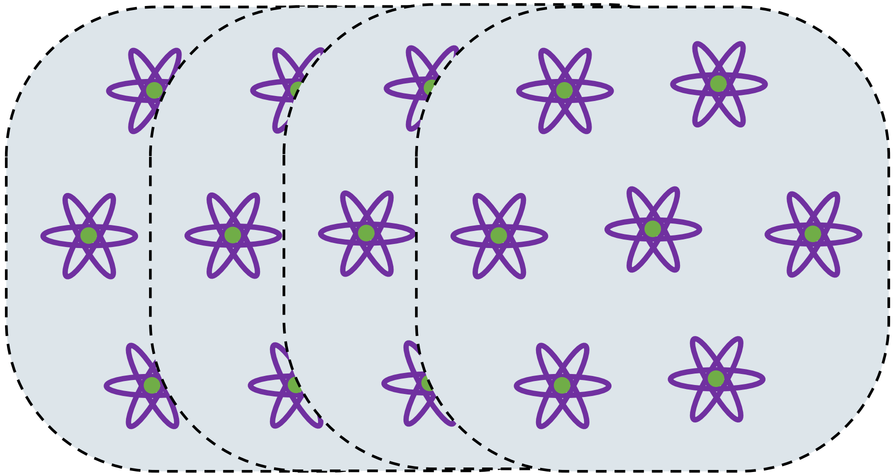
Based on joint work with

Robert Huang and John Preskill

Quantum State Certification

Lab state ρ

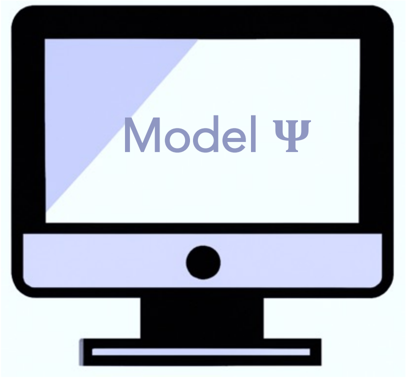


measurement data

Target state

$|\psi\rangle = \sum_{x \in \{0,1\}^n} \psi(x)|x\rangle$

$x \in \{0,1\}^n$



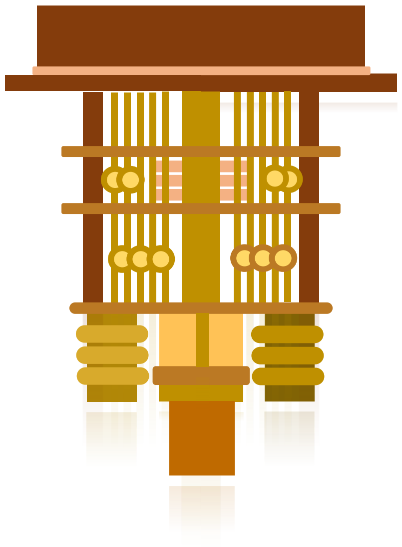
$\psi(x)$

query access to $|\psi\rangle$

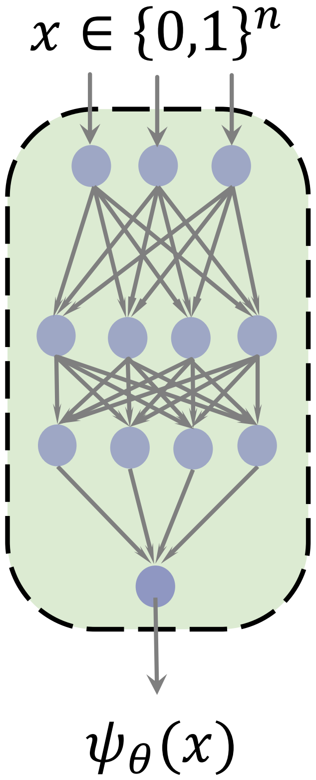
$\langle\psi|\rho|\psi\rangle$ is close to 1 or far from 1?

Applications

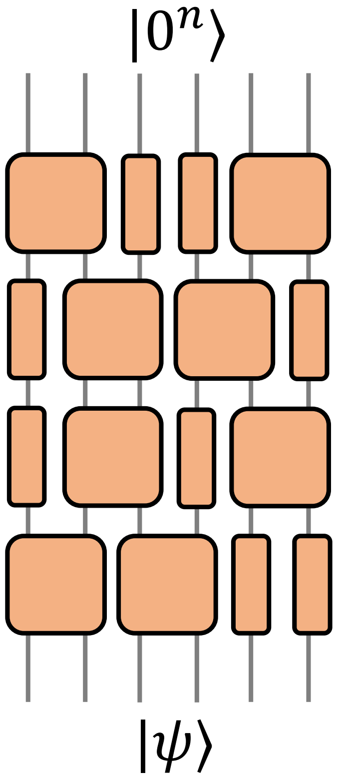
Benchmarking quantum devices



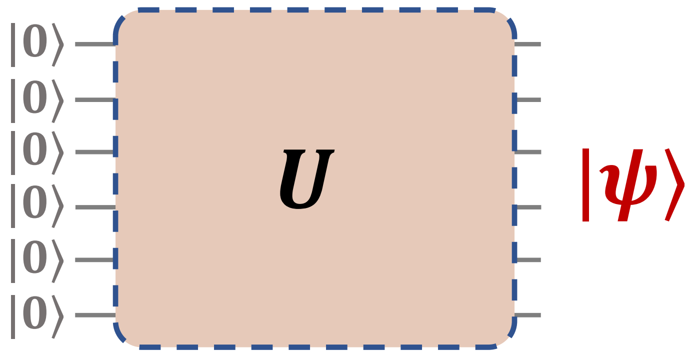
ML tomography of quantum states



Optimizing quantum circuits



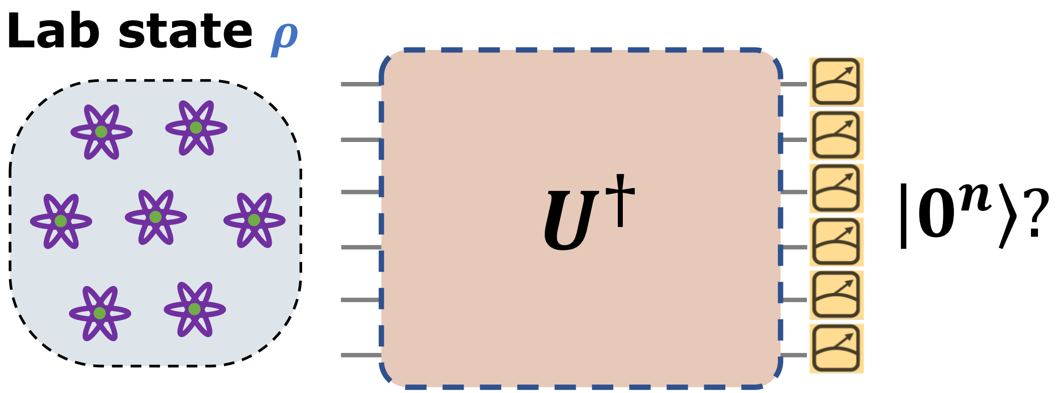
Previous approaches to state certification



Previous approaches to state certification

Measure $U^\dagger \rho U$ with $\{|0^n\rangle\langle 0^n|, \mathbf{I} - |0^n\rangle\langle 0^n|\}$
or perform SWAP test on ρ and $|\psi\rangle$

- *For general $|\psi\rangle$, requires applying highly complex U (or $U^\dagger$) s.t. $|\psi\rangle = U|0^n\rangle$*

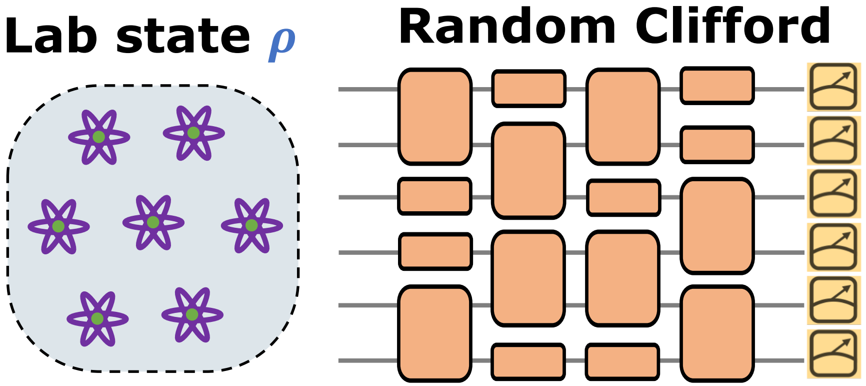


Previous approaches to state certification

Classical shadows:

Option 1: Random Clifford circuits and Z-basis measurements

- *Requires deep quantum circuits involving random gates*
- *Costly post-processing with many queries for general* $|\psi\rangle$

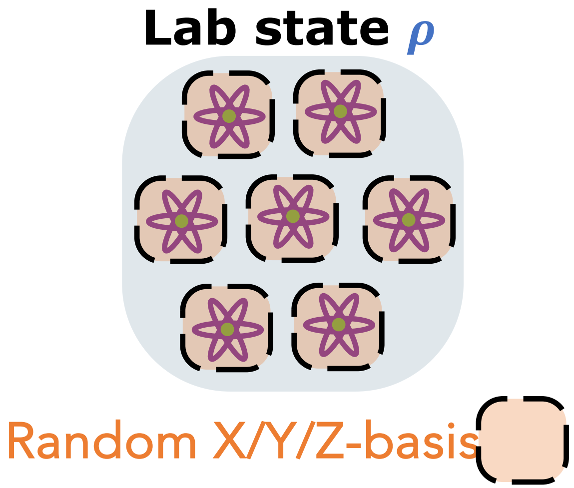


Previous approaches to state certification

Classical shadows:

Option 2: Random Pauli measurements

- *Requires* $\mathbf{exp}(\boldsymbol{n})$ *samples for general* $|\boldsymbol{\psi}\rangle$

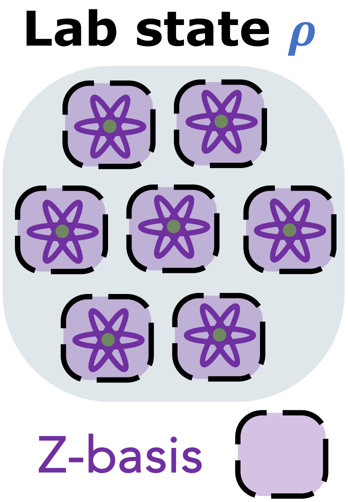


Previous approaches to state certification

Cross-Entropy Benchmarking (XEB)

$$\text{XEB} \propto \mathbb{E}_{z\sim\rho} |\langle z|\psi\rangle|^2 - 1/2^n$$

- Heuristic and can incorrectly certify when $\langle\psi|\rho|\psi\rangle \ll 1$



Previous approaches to state certification

Many other methods use Pauli measurements

But only work for special families of states $|\psi\rangle$

Including:

- Hypergraph states, output states of IQP circuits [TM18]
- Bosonic Gaussian states [AGKE15]
- Fermionic Gaussian states [GKEA18]

Previous approaches to state certification

Existing approaches to certification either

- Require deep quantum circuits
- Need exp(n) many single-qubit measurements
- Heuristic without performance guarantees
- Restricted to special families of (low-entangled) states

Can we certify quantum states with few single-qubit measurements?

Main results

For almost all n-qubit states $|\psi\rangle$, we can certify ρ is close to $|\psi\rangle$ using only $O(n^2)$ rounds of single-qubit measurements.

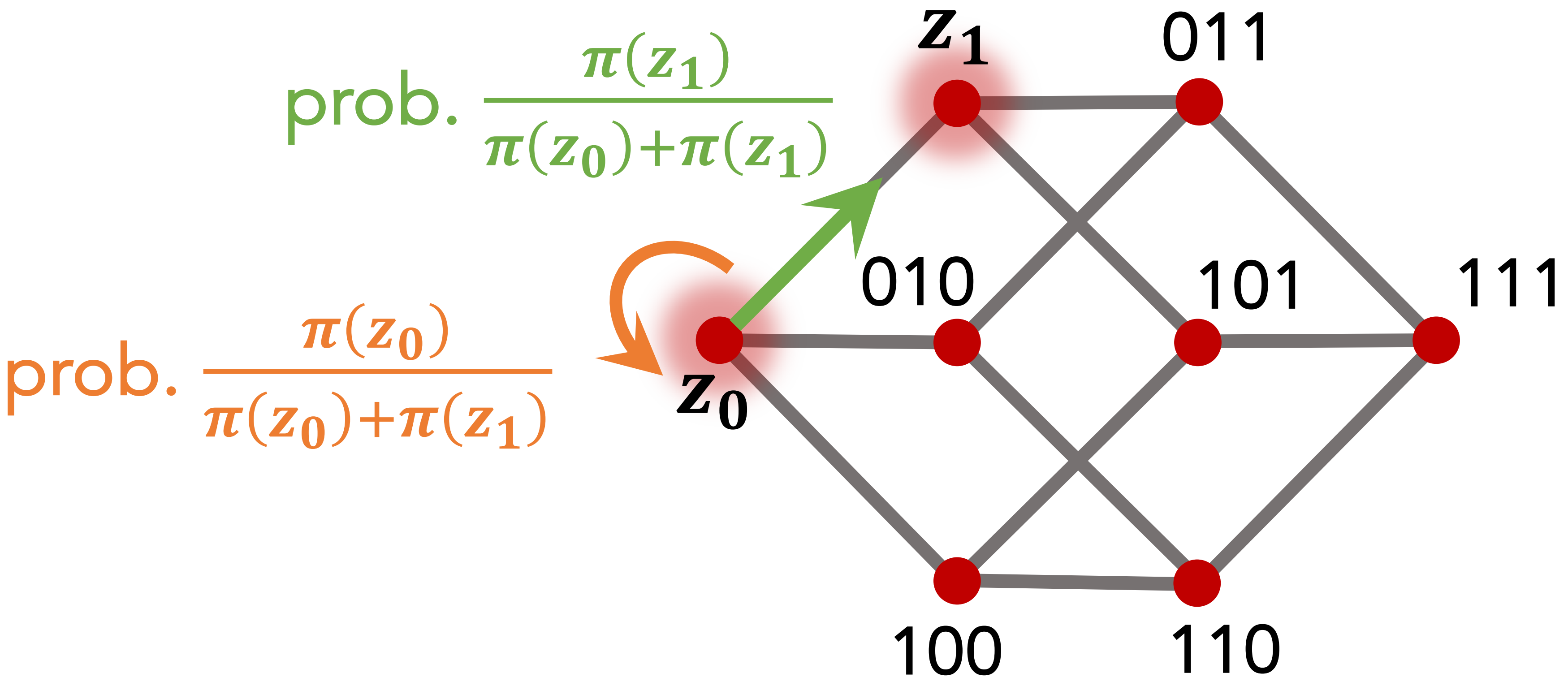
- Applies to any arbitrary ρ
- $O(n^2)$ samples even when $|\psi\rangle$ has $\exp(n)$ *circuit complexity*
- 2 queries to amplitudes of $|\psi\rangle$ in each measurement round

What about structured/specific target states?

Relaxation time

Measurement distribution: $\pi(z) = |\langle z|\psi\rangle|^2$ for $z \in \{0,1\}^n$

We think of $\pi(z)$ as stationary dist. of a random walk

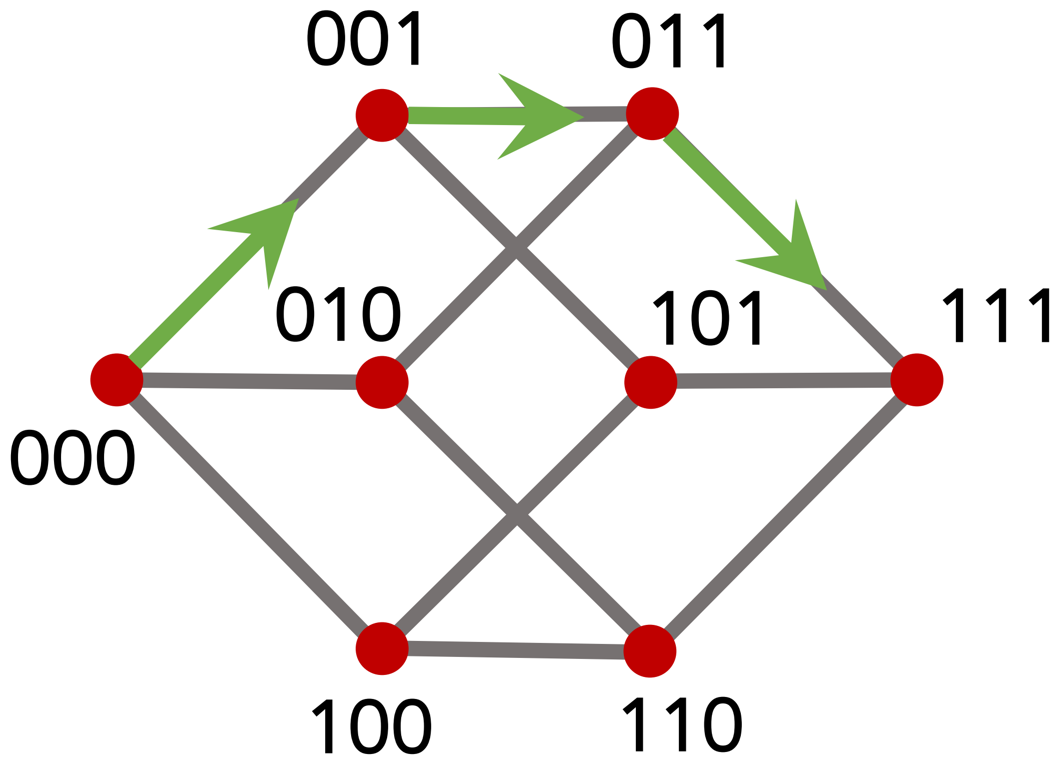


Relaxation time

Measurement distribution: $\pi(z) = |\langle z|\psi\rangle|^2$ for $z \in \{0,1\}^n$

We think of $\pi(z)$ as stationary dist. of a random walk

τ is relaxation time of random walk to stationary dist. $\pi(z)$



What about structured/specific target states?

Main results

For an $\boldsymbol{n}$-qubit state $|\boldsymbol{\psi}\rangle$ with relax. time $\boldsymbol{\tau}$, we can certify $\boldsymbol{\rho}$ is close to $|\boldsymbol{\psi}\rangle$ using $\boldsymbol{O}(\boldsymbol{\tau})$ rounds of single-qubit measurements.

- $\boldsymbol{O}(\boldsymbol{\tau}^2)$ rounds of single-qubit Pauli measurements is sufficient.

  Measurement protocol becomes offline independent of $|\boldsymbol{\psi}\rangle$.

- *We show that* $\boldsymbol{\tau} = \boldsymbol{O}(\boldsymbol{n}^2)$ *for Haar random n-qubit states*

- Also $\boldsymbol{\tau} = \mathbf{poly}(\boldsymbol{n})$ *for many structured states*

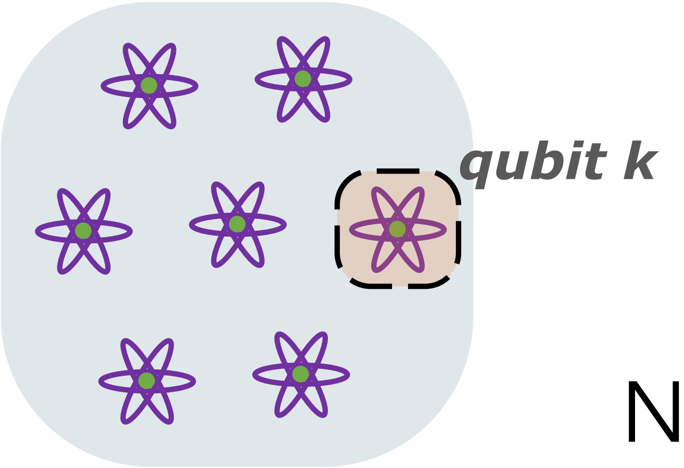
  (e.g. phase states, ground states, GHZ state)

What is the certification protocol?

One qubit at a time: compare locally, guarantee globally

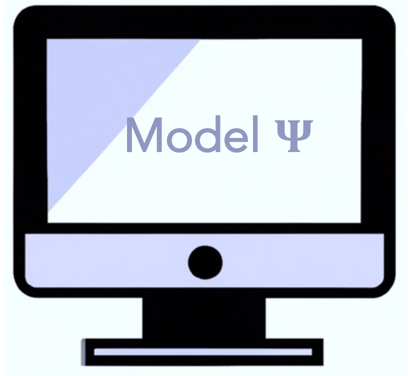
Shadow overlap protocol

Lab state $\boldsymbol{\rho}$



Target state

$|\boldsymbol{\psi}\rangle = \sum_{\boldsymbol{x} \in \{\mathbf{0},\mathbf{1}\}^n} \boldsymbol{\psi}(\boldsymbol{x})|\boldsymbol{x}\rangle$



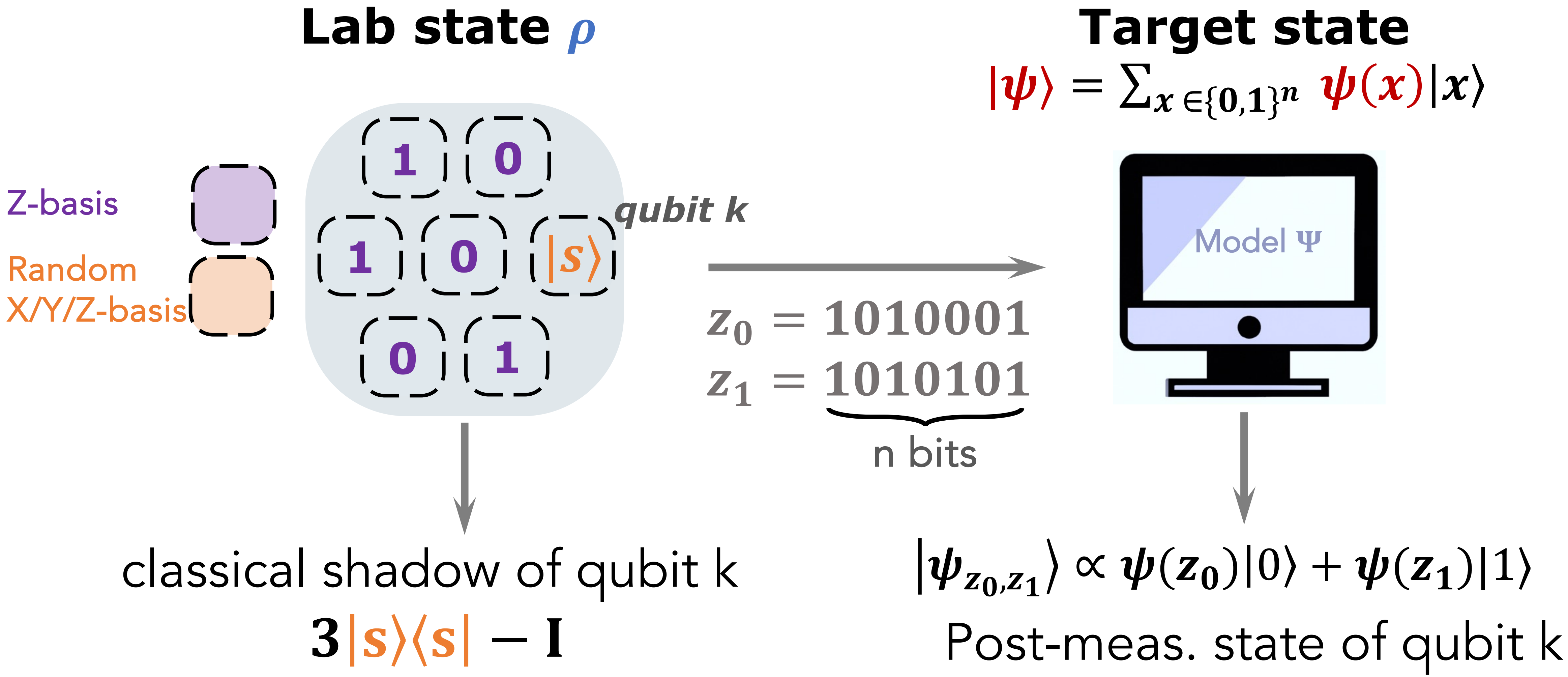
Naïve idea:

For each copy of $\boldsymbol{\rho}$, choose a qubit at random

Compare the state of that qubit between $\boldsymbol{\rho}$ and $|\boldsymbol{\psi}\rangle$

Issue: state of a single qubit $\approx$ maximally mixed

Shadow overlap protocol



Shadow overlap protocol

Predict overlap $\boldsymbol{\omega}$ of qubit k and $|\boldsymbol{\psi}_{\boldsymbol{z_0},\boldsymbol{z_1}}\rangle$

$$\boldsymbol{\omega} = \langle \boldsymbol{\psi}_{\boldsymbol{z_0},\, \boldsymbol{z_1}} | (\mathbf{3}|\mathbf{s}\rangle\langle\mathbf{s}| - \mathbf{I})\, | \boldsymbol{\psi}_{\boldsymbol{z_0},\, \boldsymbol{z_1}} \rangle$$

Shadow overlap = $\mathbb{E}[\boldsymbol{\omega}]$

classical shadow of qubit k

$\mathbf{3}|\mathbf{s}\rangle\langle\mathbf{s}| - \mathbf{I}$

$|\boldsymbol{\psi}_{\boldsymbol{z_0},\boldsymbol{z_1}}\rangle \propto \boldsymbol{\psi}(\boldsymbol{z_0})|0\rangle + \boldsymbol{\psi}(\boldsymbol{z_1})|1\rangle$

Post-meas. state of qubit k

Shadow overlap vs fidelity

Shadow overlap $\mathbb{E}[\boldsymbol{\omega}]$ accurately tracks fidelity $\langle\boldsymbol{\psi}|\boldsymbol{\rho}|\boldsymbol{\psi}\rangle$.

$$\mathbb{E}[\boldsymbol{\omega}] \geq 1-\epsilon \text{ implies } \langle\boldsymbol{\psi}|\boldsymbol{\rho}|\boldsymbol{\psi}\rangle \geq 1-\boldsymbol{\tau}\,\epsilon.$$

$$\langle\boldsymbol{\psi}|\boldsymbol{\rho}|\boldsymbol{\psi}\rangle \geq 1-\epsilon \text{ implies } \mathbb{E}[\boldsymbol{\omega}] \geq 1-\epsilon.$$

$\boldsymbol{\tau}$ as before is relaxation time of the walk induced by $|\boldsymbol{\psi}\rangle$

Proof idea

Claim: Shadow overlap $\mathbb{E}[\boldsymbol{\omega}] = \mathbf{Tr}(\boldsymbol{L}\,\boldsymbol{\rho})$

Observable $\boldsymbol{L} =$ Approximate projector onto $|\boldsymbol{\psi}\rangle\langle\boldsymbol{\psi}|$

- $\boldsymbol{L}\,|\boldsymbol{\psi}\rangle = |\boldsymbol{\psi}\rangle$
- $\langle\boldsymbol{\psi}^{\perp}|\boldsymbol{L}\,|\boldsymbol{\psi}^{\perp}\rangle \leq \mathbf{1} - \mathbf{1}/\boldsymbol{\tau}$

Relaxation time

$\boldsymbol{I} - \boldsymbol{L}$ is like a sparse Hamiltonian with ground state $|\boldsymbol{\psi}\rangle$

Proof idea

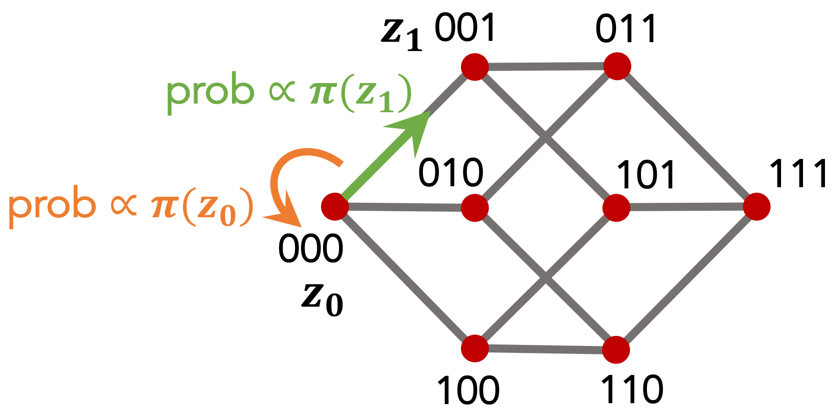
Claim: Shadow overlap $\mathbb{E}[\omega] = \mathbf{Tr}(L\,\rho)$

Observable $L =$ Approximate projector onto $|\psi\rangle\langle\psi|$

- $L\,|\psi\rangle = |\psi\rangle$
- $\langle\psi^\perp|L\,|\psi^\perp\rangle \leq 1 - 1/\tau$

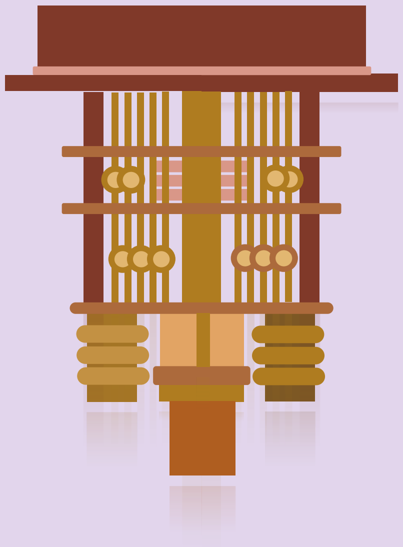
Relaxation time

$L =$ (normalized) transition matrix of a random walk

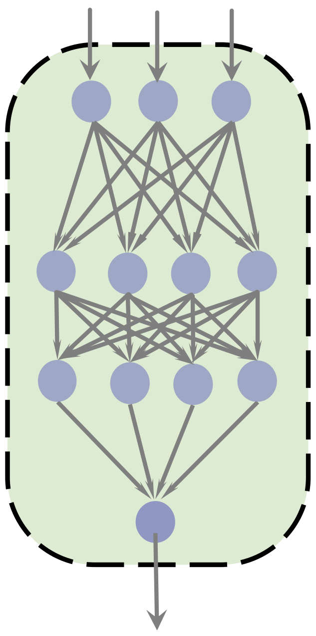


Applications

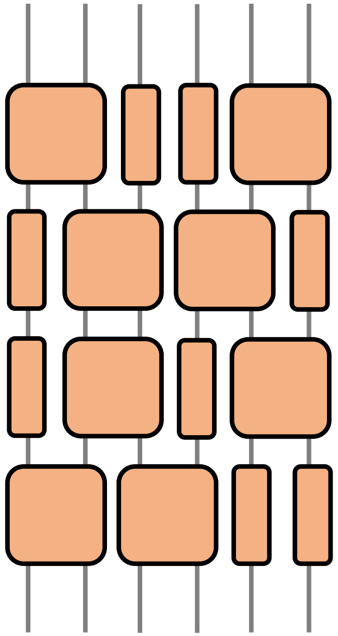
Benchmarking quantum devices



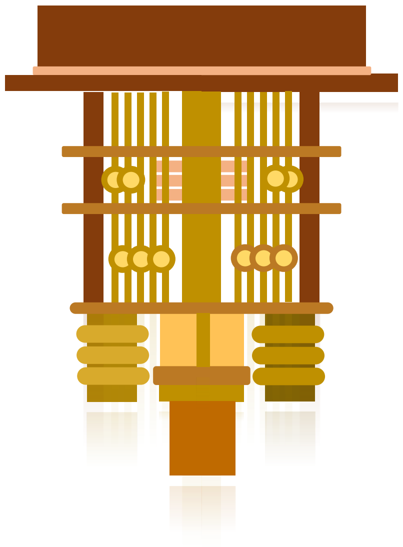
ML tomography of quantum states



Optimizing quantum circuits



Applications: Benchmarking quantum devices



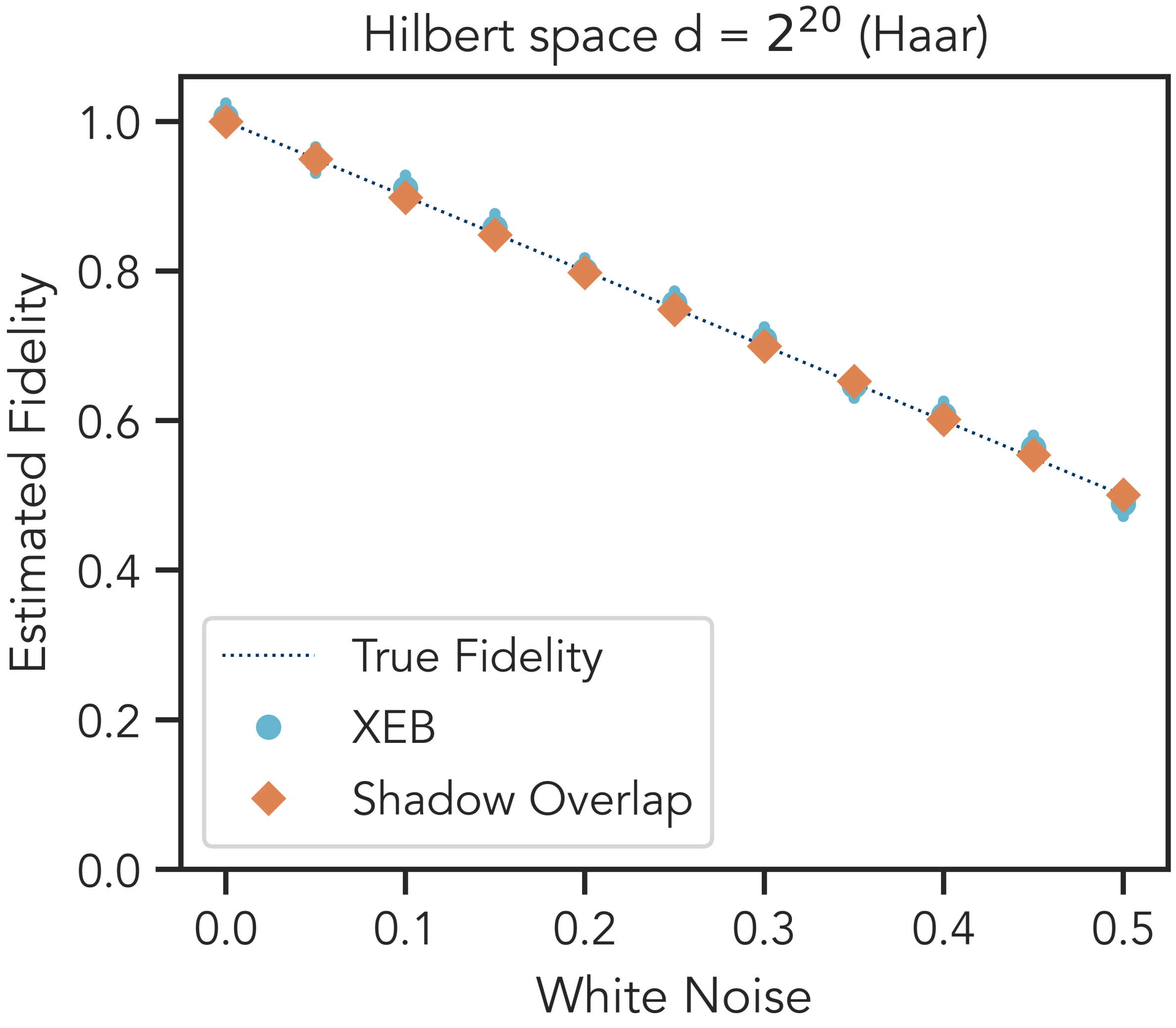
ρ is prepared by a noisy quantum device

$|\psi\rangle$ is the ideal state

Goal:
Certify $\langle\psi|\rho|\psi\rangle$ *is large*

Applications: Benchmarking quantum devices

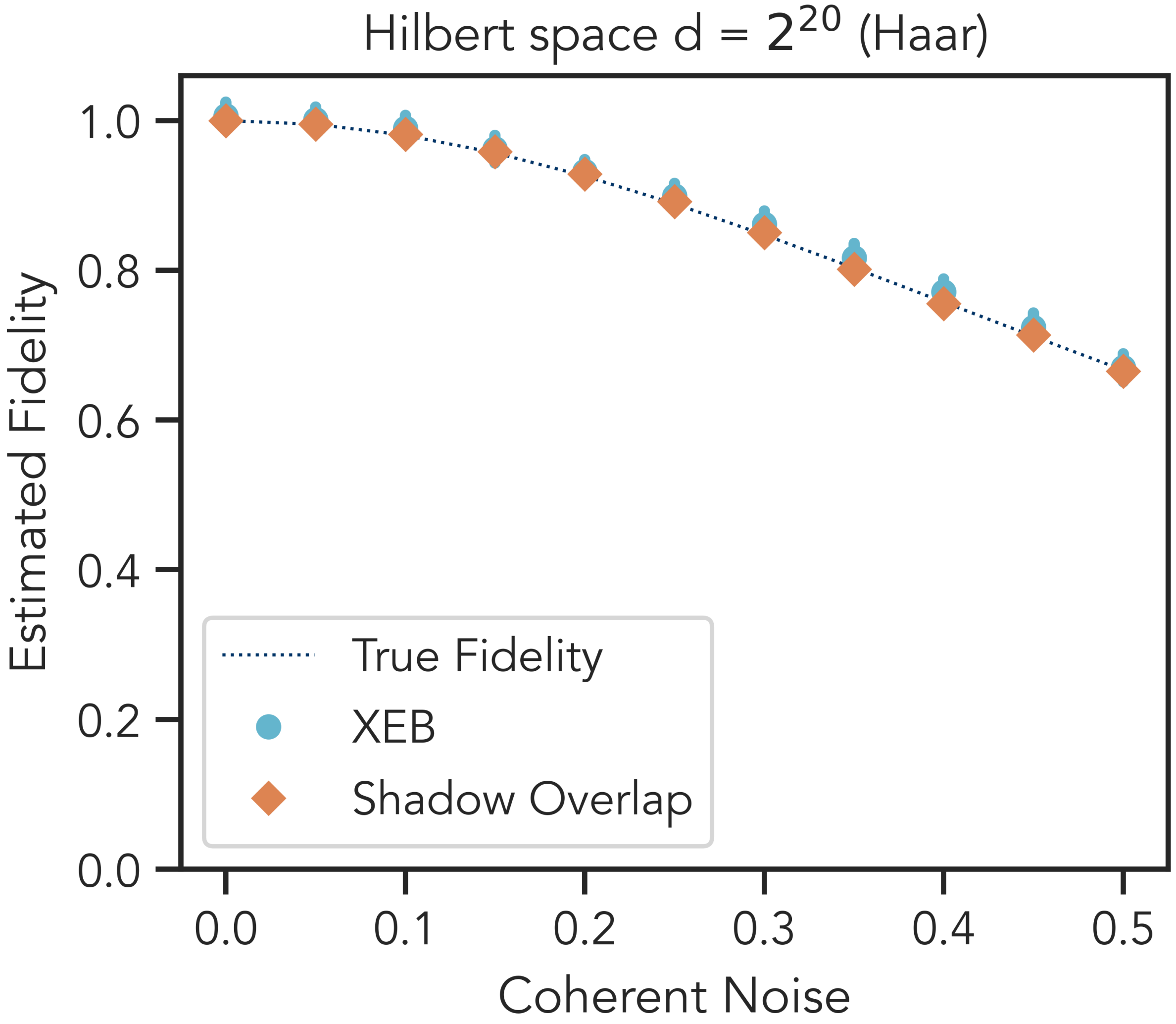
20-qubit Haar random state with white noise



* Shadow overlap normalized s.t. target state is 1, maximally mixed state is $1/2^n$

Applications: Benchmarking quantum devices

20-qubit Haar random state with coherent noise



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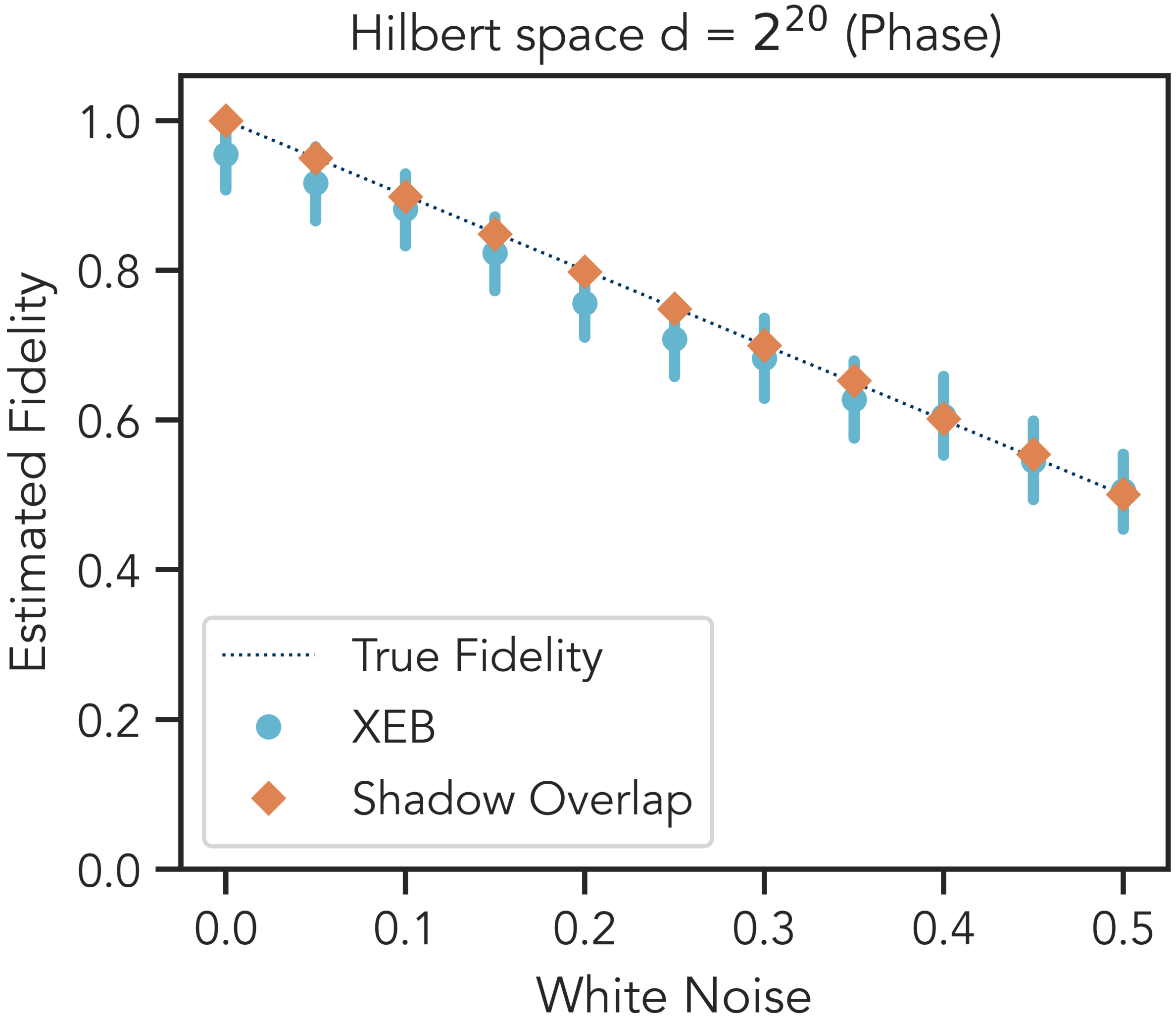
Applications: Benchmarking quantum devices

20-qubit random phase state
with white noise

$$|\psi\rangle = U_{\text{phase}} \otimes_{k=1}^{20} |\psi_i\rangle$$



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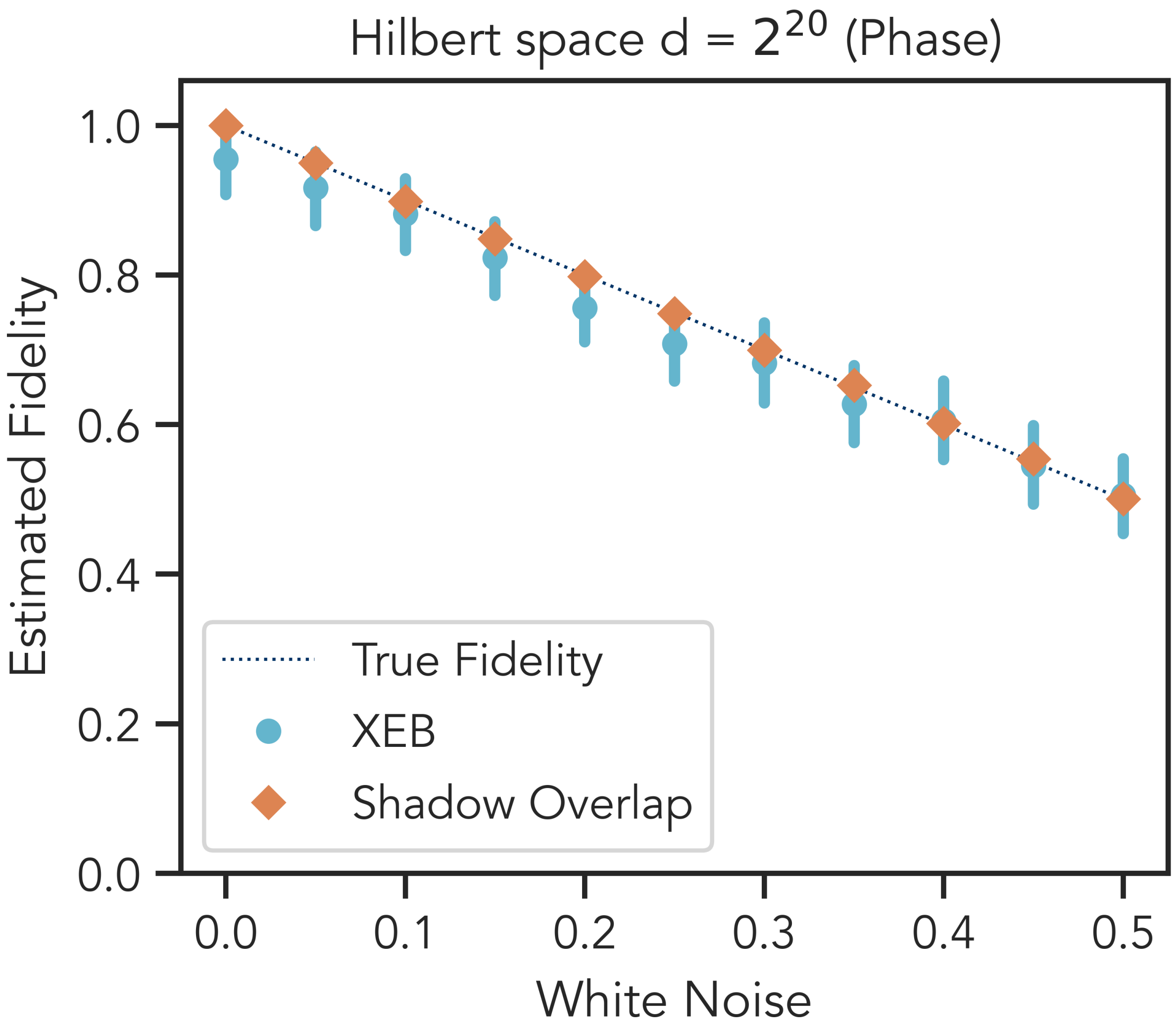
Applications: Benchmarking quantum devices

20-qubit random phase state
with white noise

$$|\psi\rangle = \boldsymbol{U}_{\mathbf{phase}} \bigotimes_{k=1}^{20} |\boldsymbol{\psi_i}\rangle$$

$$U_{\text{phase}} = \text{diag}(e^{i\phi_1}, \dots, e^{i\phi_{2^n}})$$

$$|\psi_i\rangle = \cos(\theta_i)|0\rangle + \sin(\theta_i)|1\rangle$$



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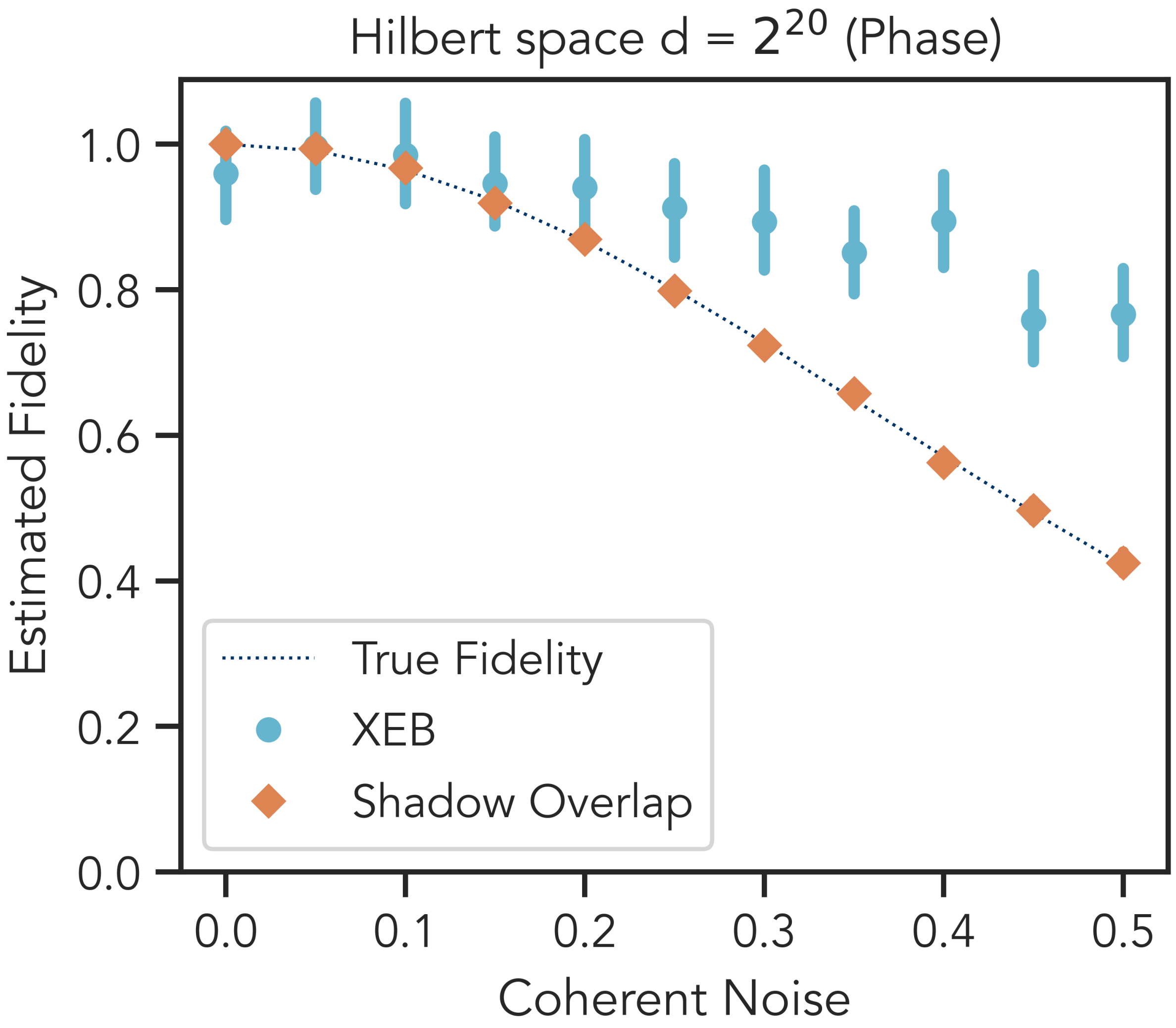

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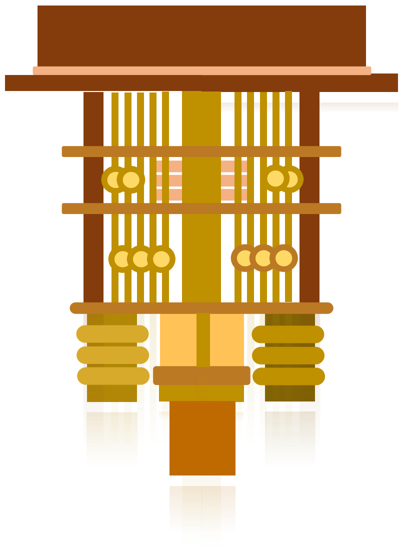
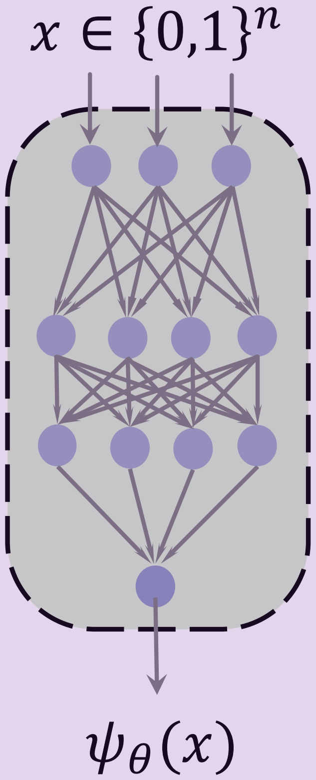
Applications

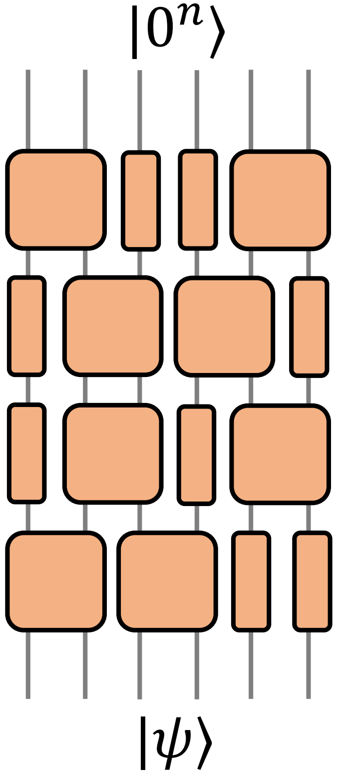
Benchmarking quantum devices



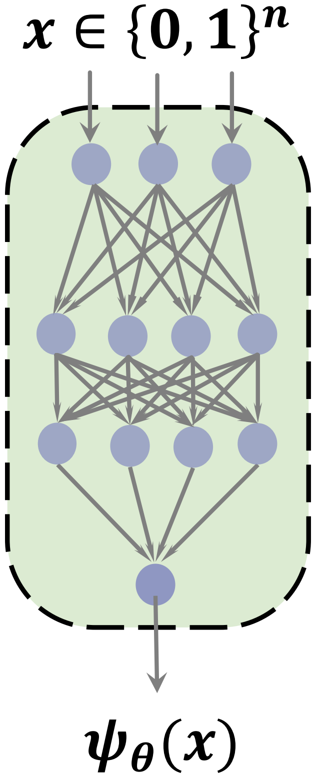
ML tomography of quantum states



Optimizing quantum circuits



Applications: ML tomography of quantum states



ρ is an unknown quantum state

$|\psi_\theta\rangle$ is an ML model with parameters θ
(a.k.a. neural quantum state)

Goal:
Train/Certify θ^* such that $\langle\psi_{\theta^*}|\rho|\psi_{\theta^*}\rangle$ *is large*

Applications: ML tomography of quantum states

- Learning random binary phase states with n=120 :

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{\phi(x)} |x\rangle$$

Highly entangled state for random $\phi(x) \in \{0, 1\}$

- Train and certify neural net using shadow overlap

Applications: ML tomography of quantum states

- Learning random binary phase states with n=120 :

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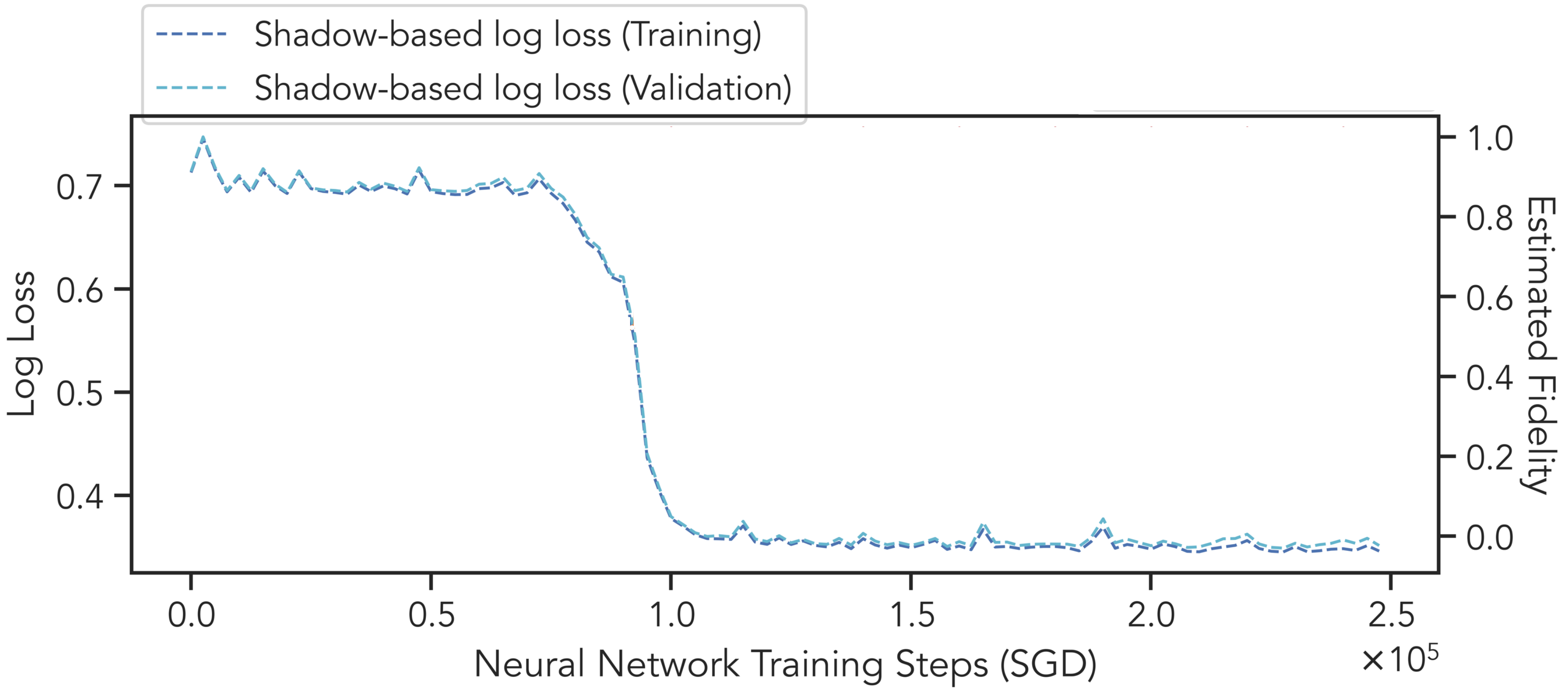

Applications: ML tomography of quantum states

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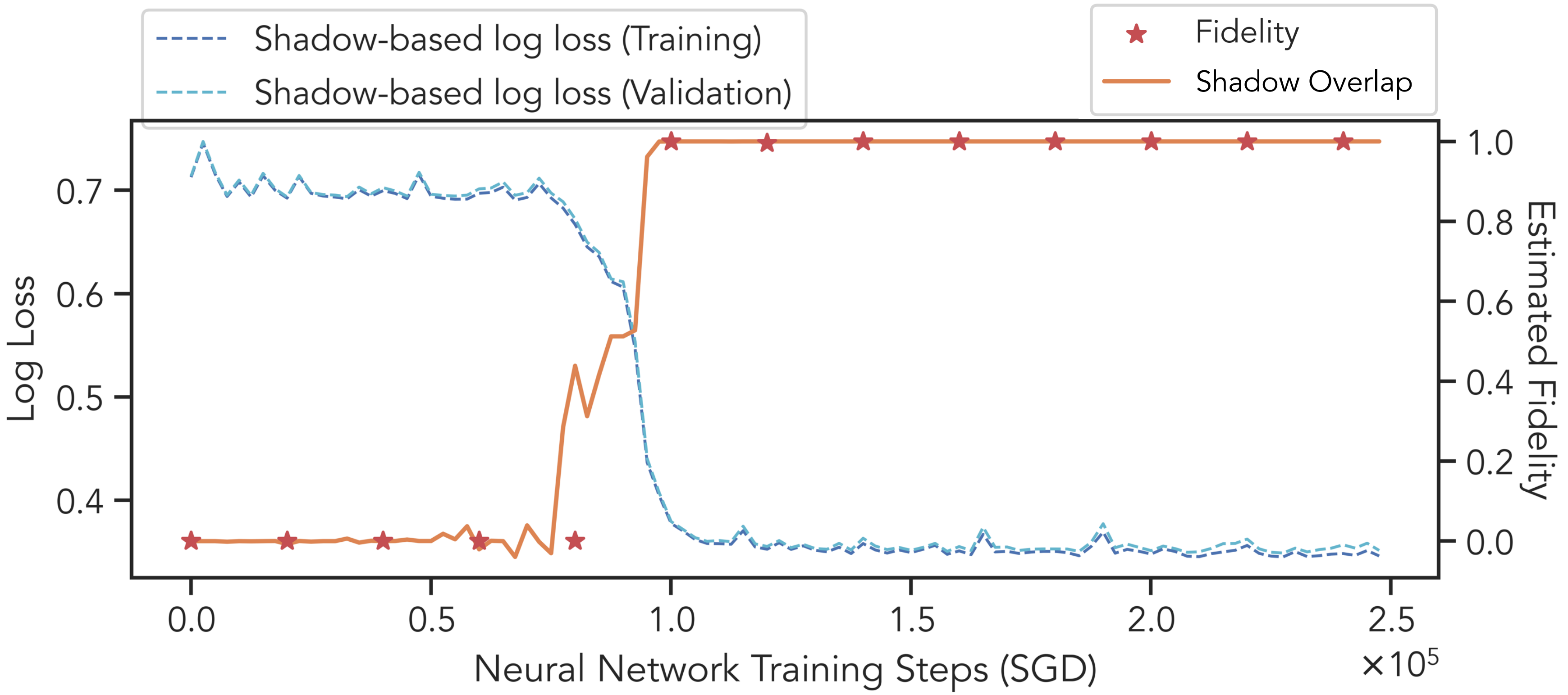

Applications: ML tomography of quantum states

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$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{\phi(x)} |x\rangle$$

Highly entangled state for random $\phi(x) \in \{0,1\}$

- Can estimate non-local properties that need $2^{O(n)}$ single-qubit meas.

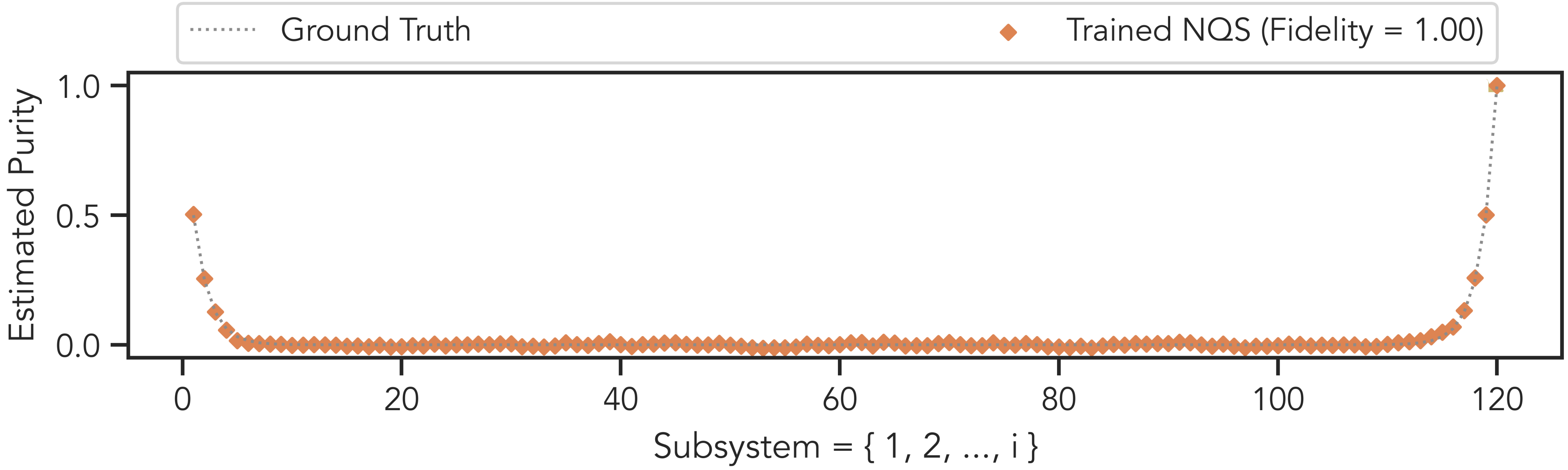
Applications: ML tomography of quantum states

- Learning random binary phase states with n=120 :

$$\frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} (-1)^{\phi(x)} |x\rangle$$

Highly entangled state for random $\phi(x) \in \{0, 1\}$

- Can estimate purity $\mathrm{tr}(\rho_A^2)$ that needs $2^{O(|A|)}$ single-qubit meas.



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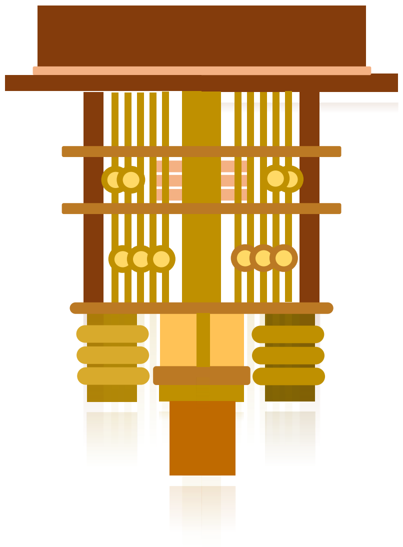

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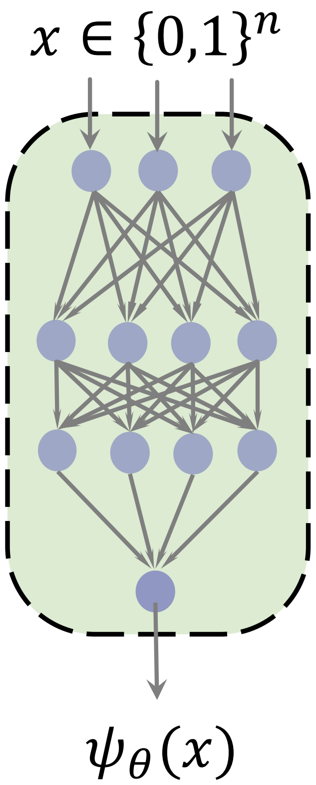
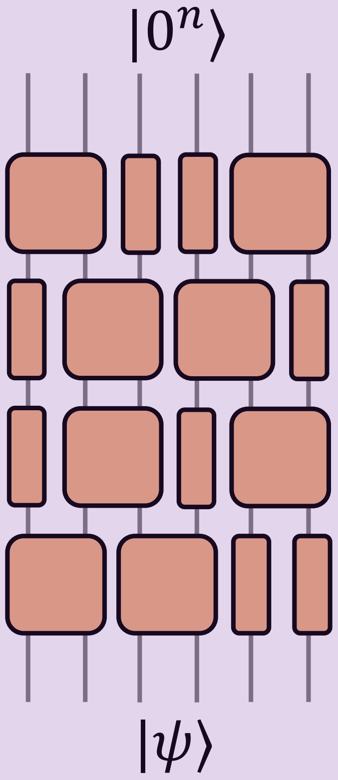
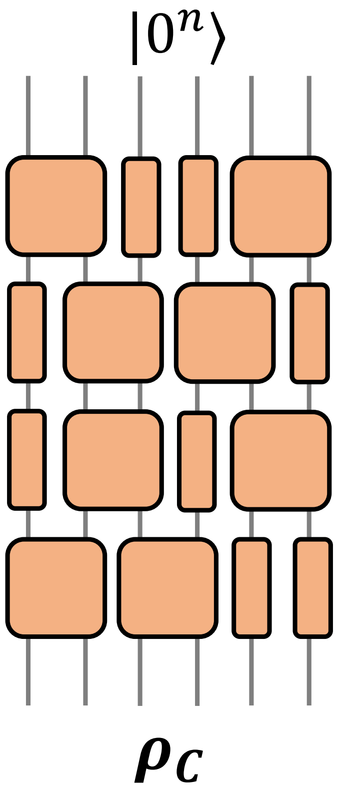


Optimizing quantum circuits



Applications: Optimizing quantum circuits



ρ_C is prepared by parametrized circuit C

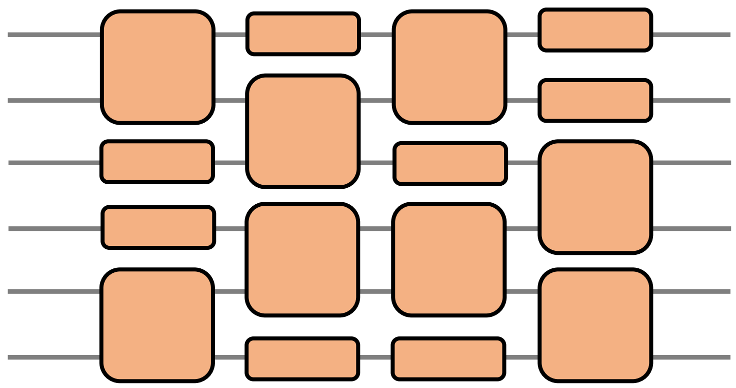
$|\psi\rangle$ is a target state

Goal:
Optimize circuit to prepare $|\psi\rangle$: $\max_C \langle\psi|\rho_C|\psi\rangle$

Applications: Optimizing quantum circuits

- We consider preparing an n-qubit MPS with *H, CZ, T gates*

- State is output of *IQP circuit + T gates*



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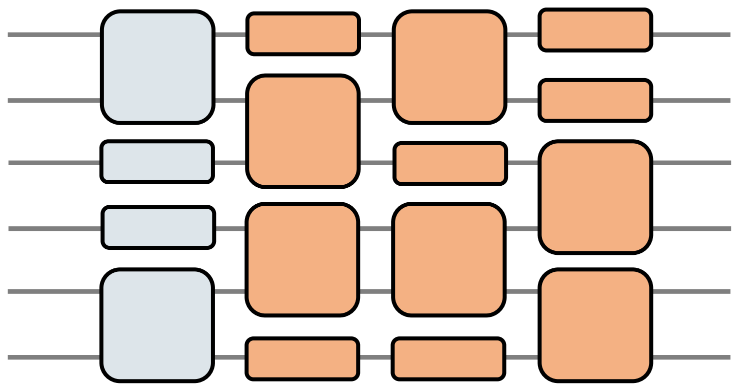

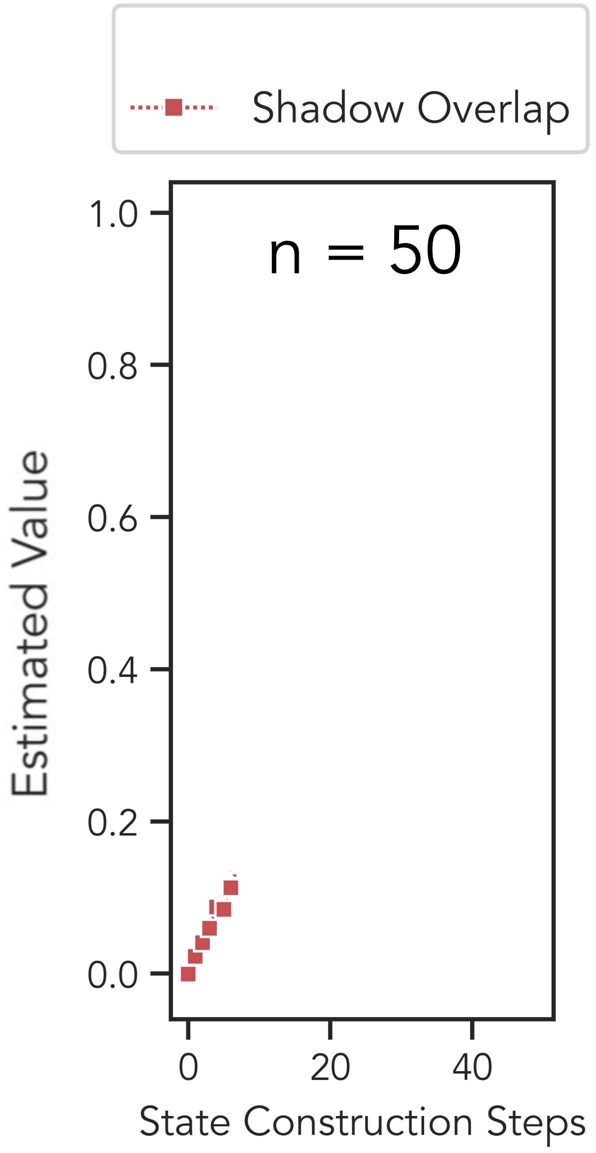

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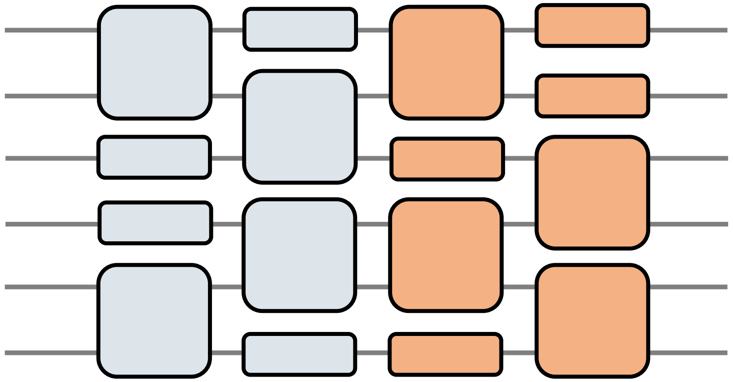

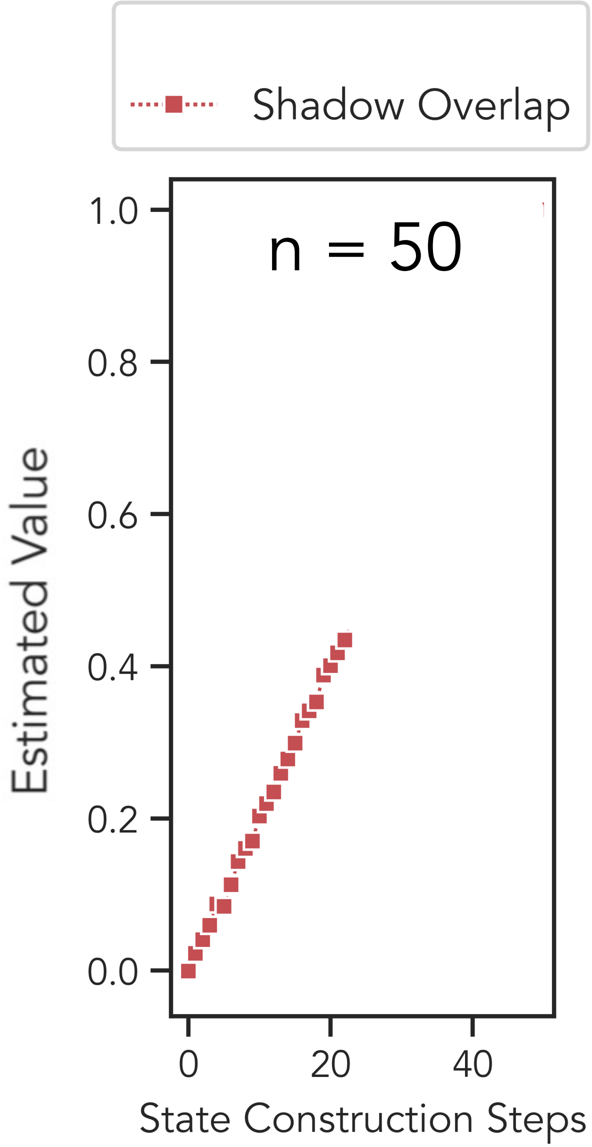

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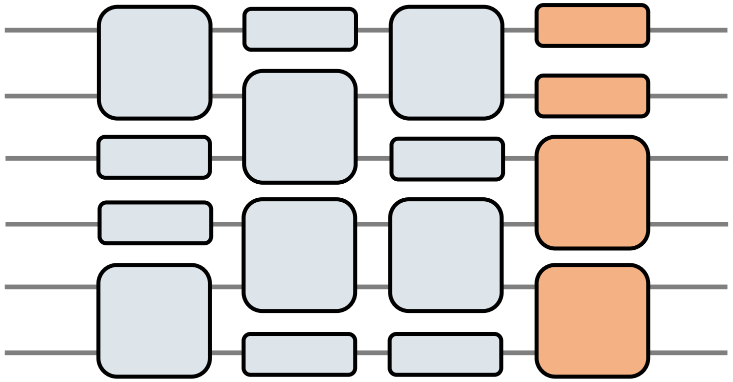

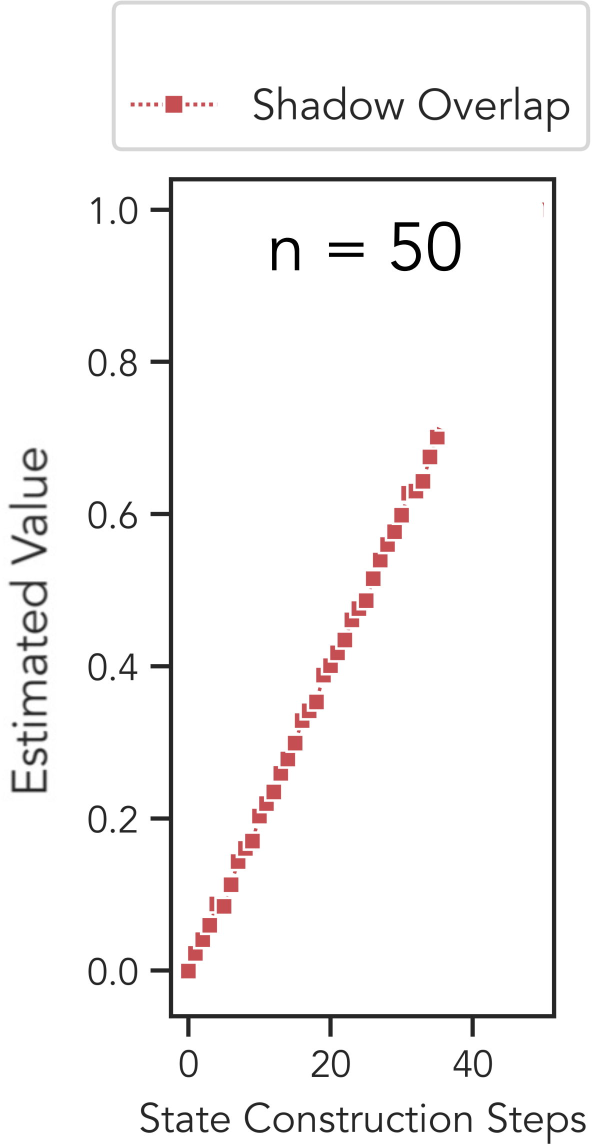

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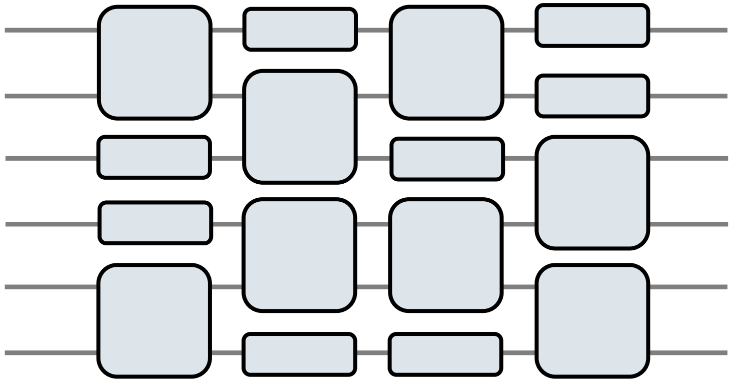

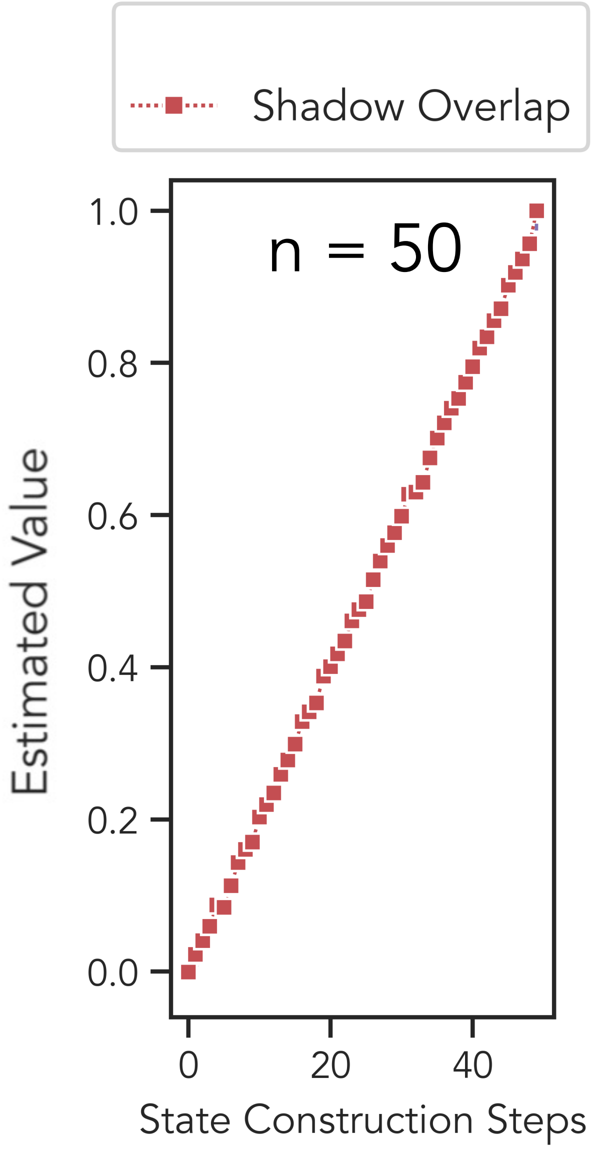

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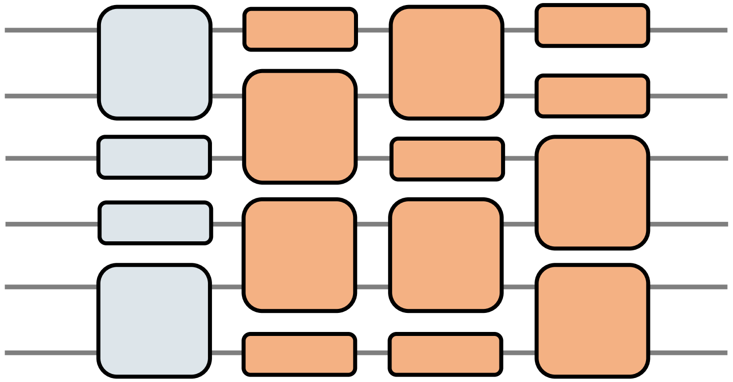

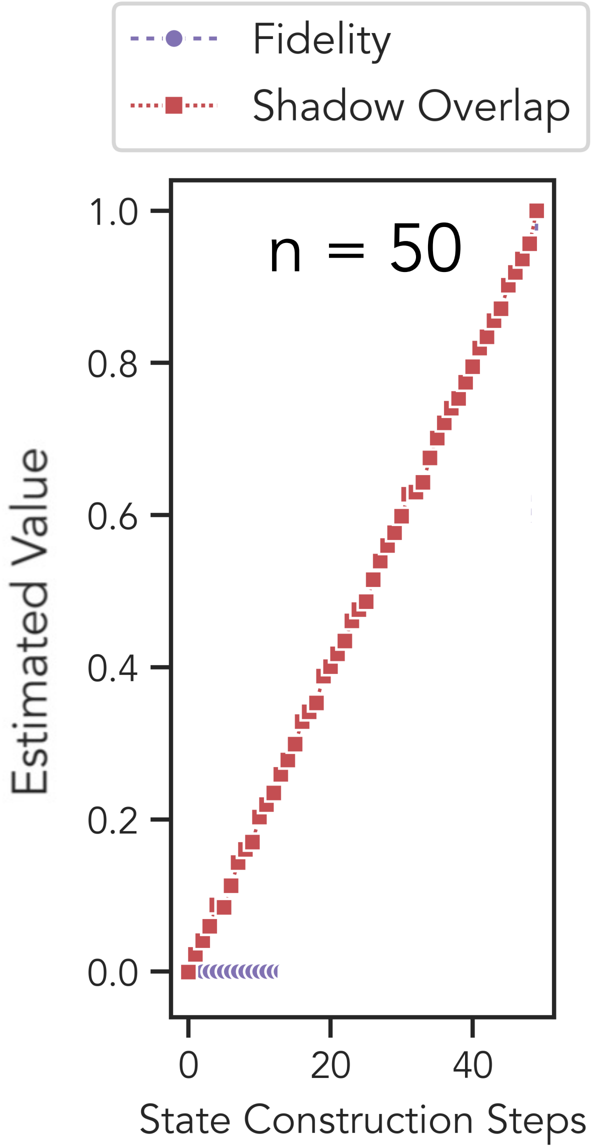

Applications: Optimizing quantum circuits

- We consider preparing an n-qubit MPS with *H, CZ, T gates*

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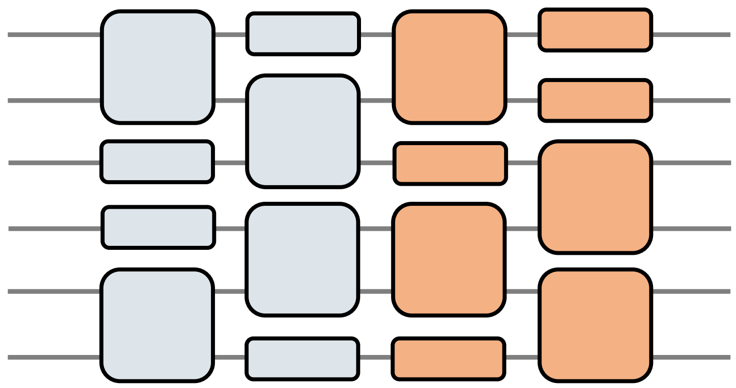

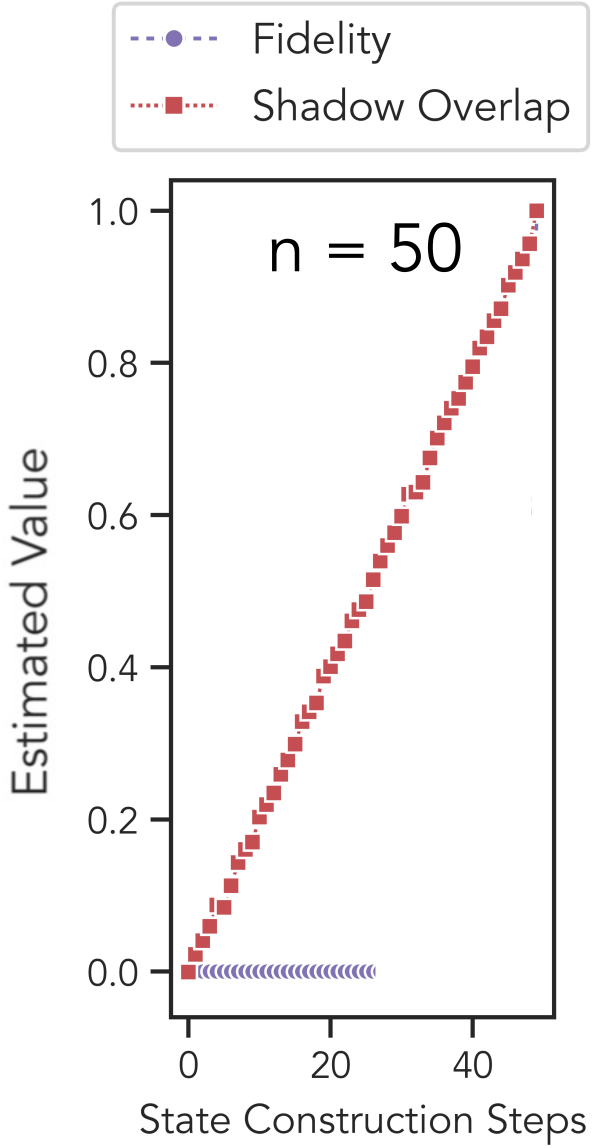

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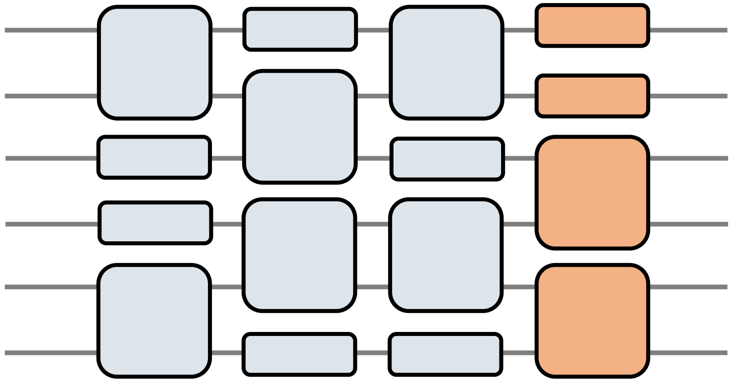

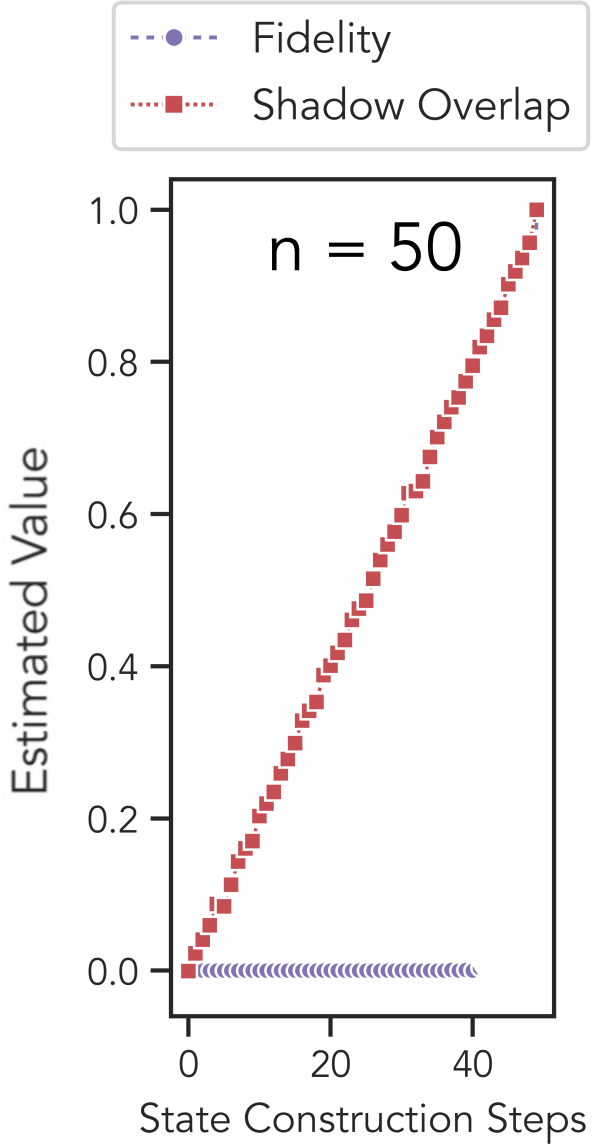

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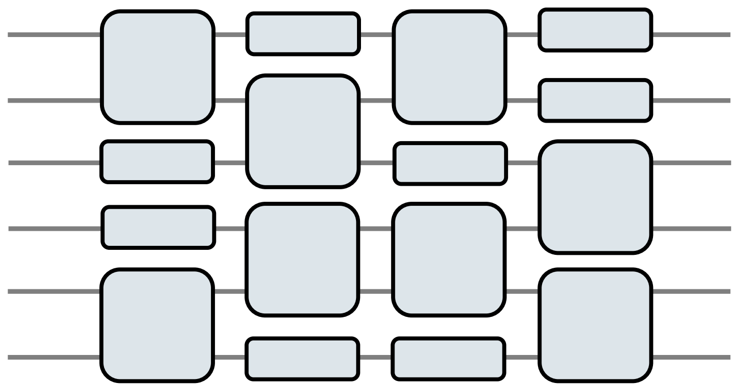
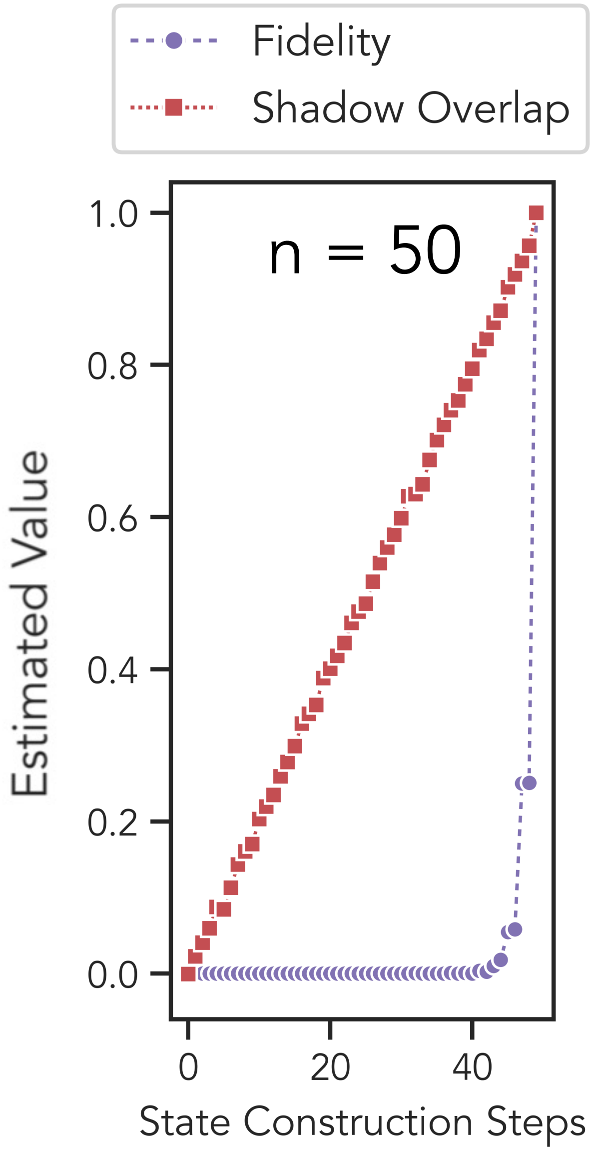
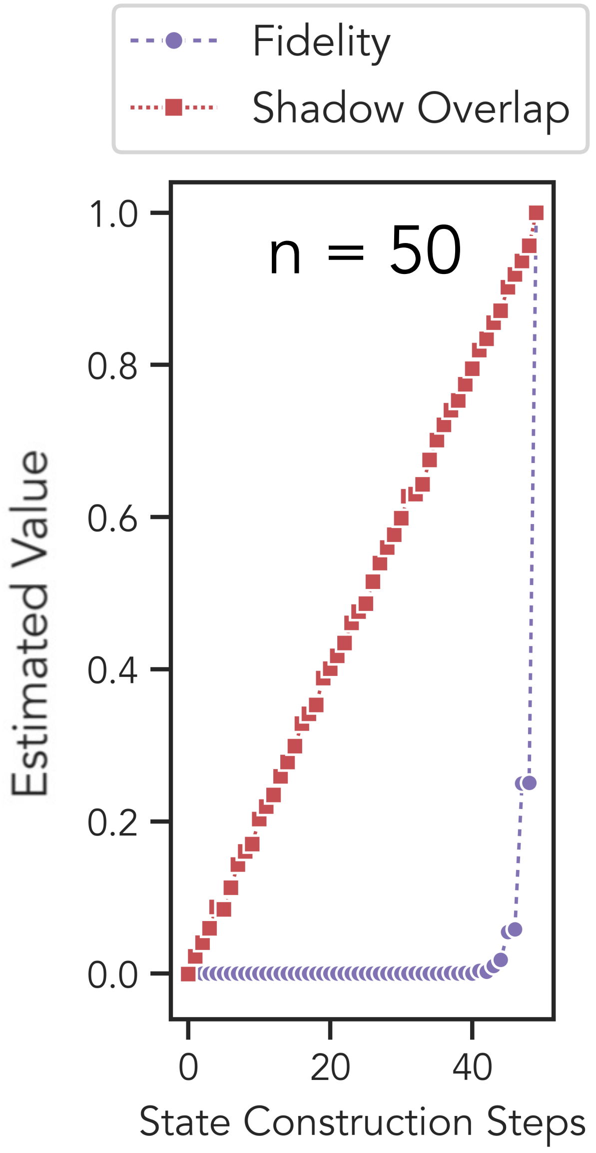




Applications: Optimizing quantum circuits

- We consider preparing an n-qubit MPS with *H, CZ, T gates*
- State is output of *IQP circuit + T gates*
- *Fidelity remains zero for most steps before growing rapidly (barren plateau pheno.)*
- *Shadow overlap acts like Hamming distance*

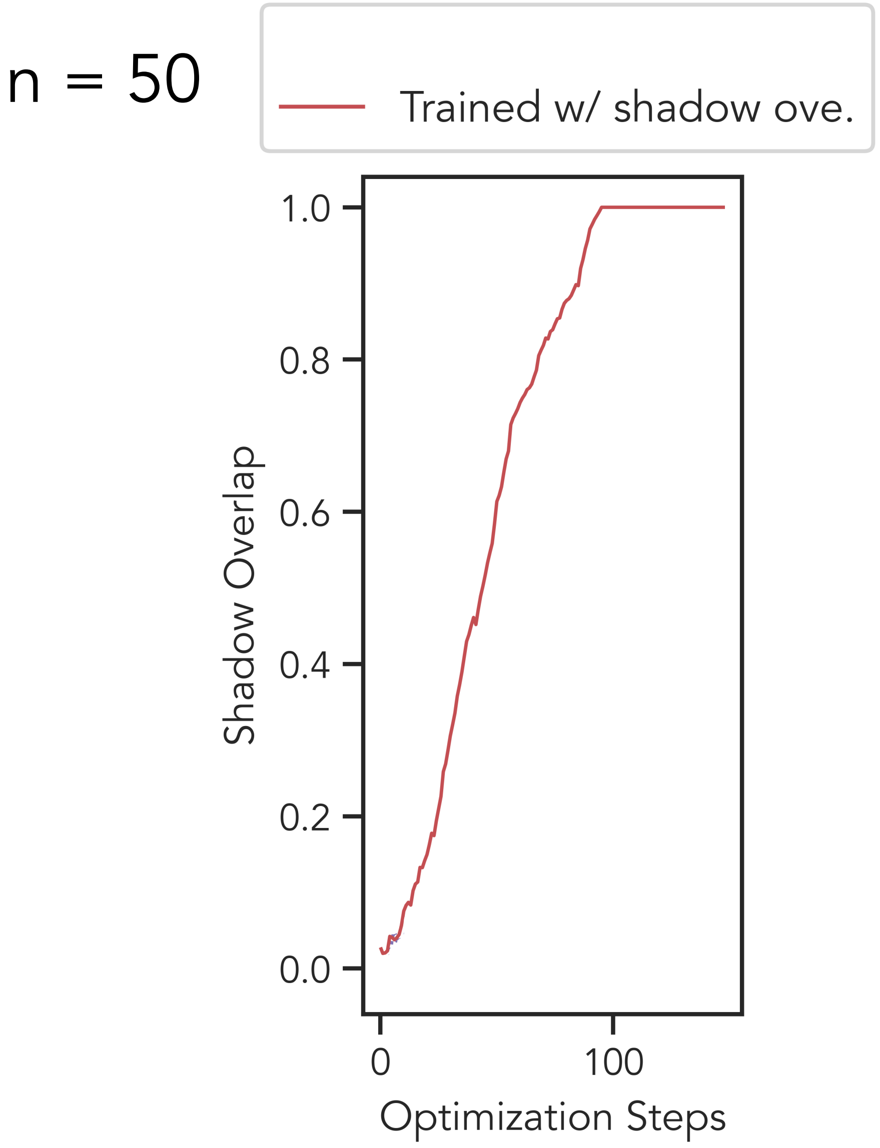


Applications: Optimizing quantum circuits

Monte-Carlo circuit optimization

with shadow overlap & fidelity

for preparing 50-qubit MPS



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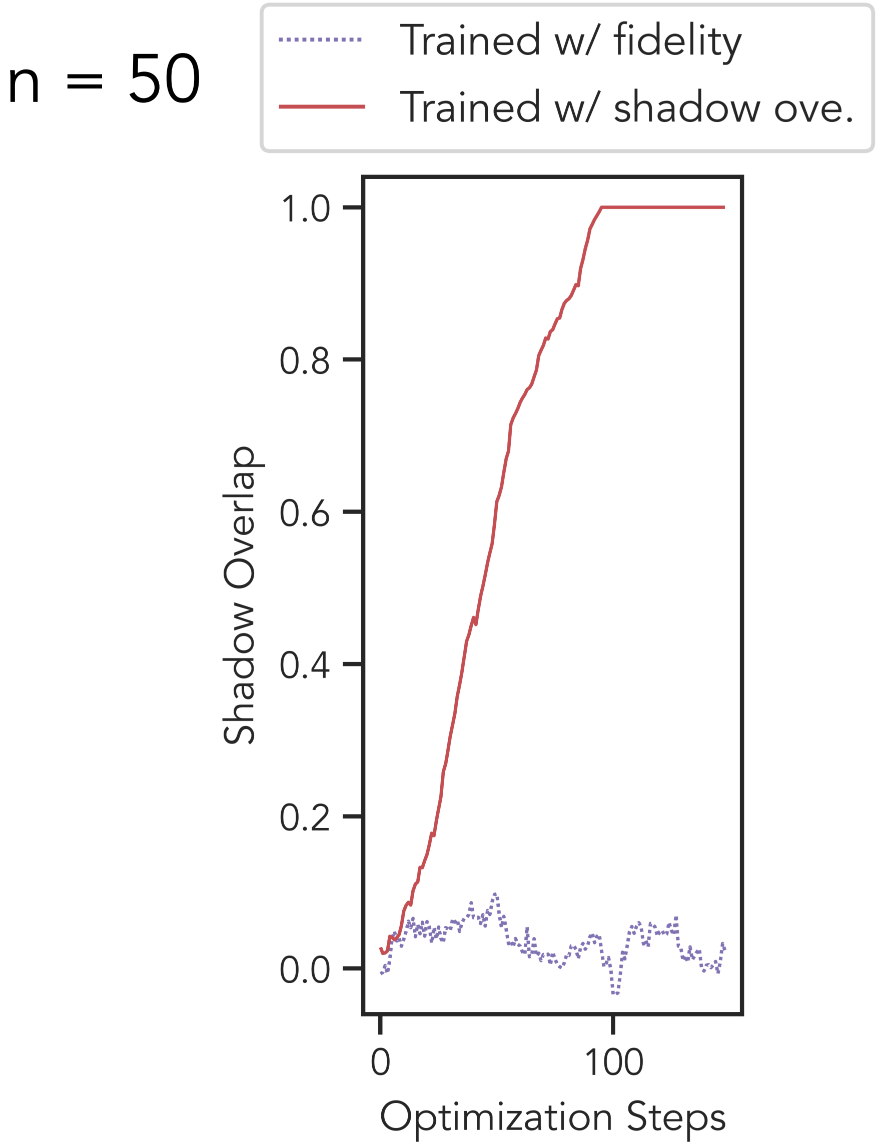
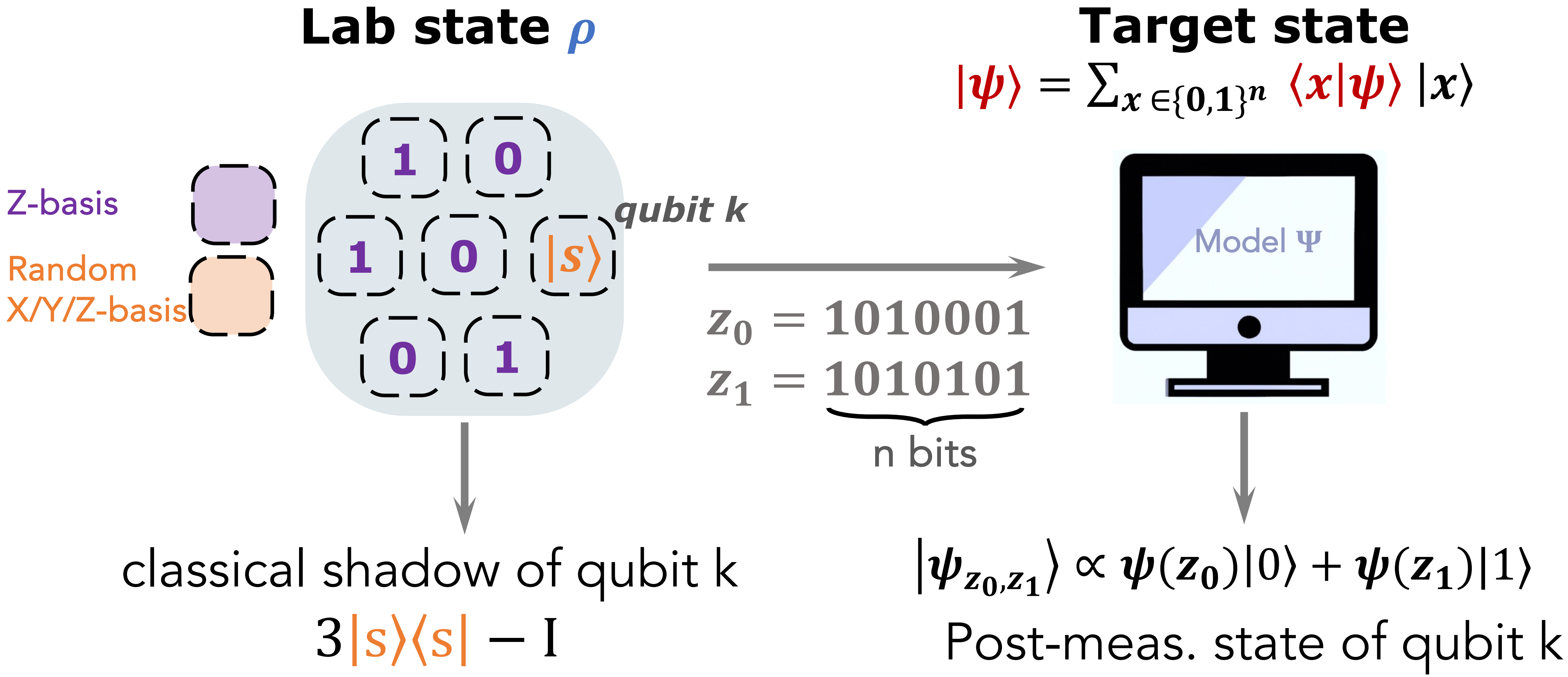
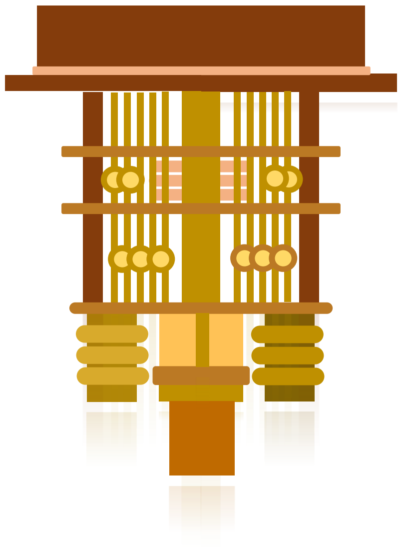


Shadow overlap protocol

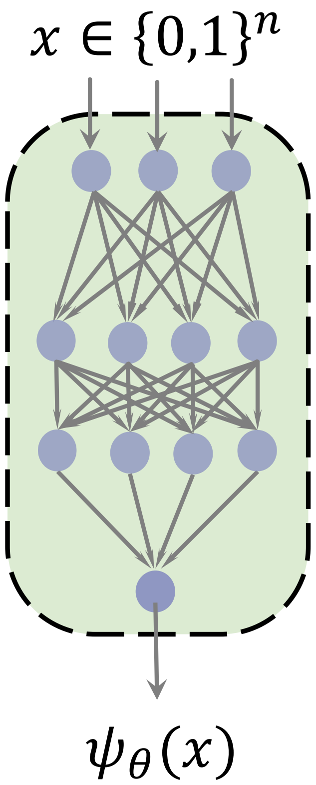


Applications

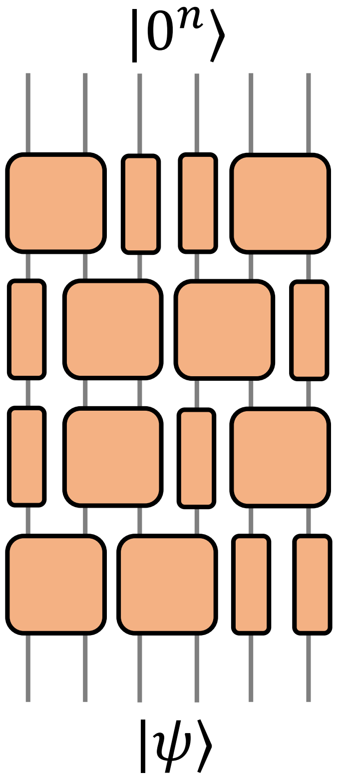
Benchmarking quantum devices



ML tomography of quantum states



Optimizing quantum circuits



Open questions

- Which families of states have fast relaxation time $\boldsymbol{\tau \leq \mathrm{poly}(n)}$?
- Can we certify all quantum states with few single-qubit measurements?
- Which states admit efficient access to their amplitudes $\boldsymbol{x \mapsto \psi(x)}$?
  e.g. via classical neural networks?

Thank you!

Certifying almost all quantum states with few single-qubit measurements

Mehdi Soleimanifar (Caltech)

Based on joint work with

Robert Huang and John Preskill